Essays on ratings, risk and relative consumption

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Contents

Abstract

This cumulative dissertation investigates the reasons for and macroeconomic consequences of borrowing constraints on three types of agents: governments, firms and households.

In the first paper, governments face borrowing constraints due to high interest rates on sovereign debt. In an empirical analysis, the interaction between these interest rates and sovereign debt ratings are examined. In such a setting, multiple equilibria can occur. There is a 'good' equilibrium with low interest rates and good ratings which is locally stable. A second, unstable equilibrium marks a threshold beyond which a country is pulled in a downward spiral of ever higher interest rates and bad ratings. This leads eventually to a third and stable equilibrium, namely, default. Rating agencies could play a major role in determining the type of equilibrium into which a country settles, in particular, for countries with weak economic fundamentals.

The second paper develops a general equilibrium model with borrowing constrained firms. Firms use equity and loans to finance the amount of capital needed for production. Idiosyncratic productivity shocks and risk premia due to financial frictions, limit the firm's access to loans. The model is used to analyse how different types of shocks affect the stationary size distribution of firms and thereby, aggregate output.

The third paper deals with borrowing constrained households. In the scholarly literature, this well established incomplete market setting has been extensively used to analyse the wealth distribution. This paper extends the benchmark model by relative consumption behaviour: how does the wealth distribution change if households value not only their own level of consumption, but relate it to others around them? In addition to this question, welfare optimizing tax policies are discussed.

Zusammenfassung

Diese kumulative Dissertation befasst sich mit den Ursachen und makroökonomischen Auswirkungen von Kreditbeschränkungen auf drei verschiedene ökonomische Akteure: Staaten, Firmen und Individuen.
Die erste Arbeit untersucht Kreditbeschränkungen auf Staatenebene. In einer empirischen Analyse wird die Interaktion zwischen Zinsen und Ratings auf Staatsschulden untersucht. Die gegenseitige Abhängigkeit sowie nicht lineare Zusammenhänge bewirken, dass multiple Gleichgewichte entstehen können. Es existiert ein 'gutes' Gleichgewicht, welches durch lokale Stabilität, niedrige Zinsen und gute Ratings gekennzeichnet ist. Ein zweites, instabiles Gleichgewicht markiert einen wichtigen Schwellenwert. Wenn einmal überschritten, wird ein Land in eine Abwärtsspirale aus immer höheren Zinsen und schlechteren Ratings gesogen. Dies endet schlussendlich in einem dritten, stabilen Gleichgewicht, dem Bankrott. Ratingagenturen könnten gerade bei ökonomisch schlecht aufgestellten Ländern eine entscheidende Rolle spielen, in welches Gleichgewicht sich ein Staat bewegt.
In der zweiten Arbeit wird ein gesamtwirtschaftliches Gleichgewichtsmodell mit kreditbeschränkten Firmen vorgestellt. Firmen können das für die Produktion benötigte Kapital sowohl mit Eigen- als auch mit Fremdkapital finanzieren. Aufgrund individueller Produktivitätsschocks und Finanzfriktionen sind Firmen in ihrer Fremdkapitalaufnahme beschränkt, da Banken Risikoprämien verlangen. Das Modell wird angewandt, um die Auswirkungen verschiedener gesamtwirtschaflicher Schocks auf die Grössenverteilung von Firmen und die aggregierte Wirtschaftsleistung zu analysieren.
Im letzten Teil der Dissertation liegt der Fokus auf kreditbeschränkten Individuen beziehungsweise Haushalten. Die vorliegende Arbeit erweitert das Standardmodell um relatives Konsumverhalten. Das heisst, Individuen wertschätzen nicht nur ihren eigenen Konsum, sondern auch, wie dieser in Relation zu dem anderer Individuen steht. In einem gesamtwirtschaftlichen Gleichgewichtsmodell wird untersucht, ob sich beobachtete Vermögensverteilungen so besser erklären lassen. Ausserdem werden wohlfahrtsoptimierende Steuersysteme ermittelt und mit bisherigen Erkenntnissen verglichen.

Introduction

This is a cumulative dissertation. It consists of three, largely distinct macroeconomic contributions in terms of content and methodology. A common denominator with respect to the content of this thesis is the analysis of borrowing constraints in its broadest sense. Each contribution discusses either reasons for or consequences of borrowing constraints with respect to three different types of agents. The first chapter focuses on governments. The second on firms and the third on households.

The first chapter is entitled "Rating agencies, self-fulfilling prophecy and multiple equilibria? An empirical model of the European sovereign debt crisis 2009 - 2011" and is co-authored with Manfred Gärtner from the University of St. Gallen. This chapter presents an empirical application of a simplified version of the sovereign debt crisis model proposed by Calvo (1988). In this theoretical contribution interactions between interest rates and probabilities of default lead to multiple equilibria. Interest rates rise with default probabilities since they contain a risk premium that covers for the expected loss. But also default probabilities rise with interest rates since they worsen a country's future debt situation. Three equilibria emerge. The first, 'good' equilibrium is stable and characterized by a low probability of default and low interest rates due to a minor risk premium. Another stable equilibrium marks a certain default. In this case, the probability of default is by definition one and the corresponding risk premium and interest rate is infinite. A third equilibrium lies in between. This one is unstable and marks a vital threshold. Adaptive behaviour either leads to certain default or back to the good equilibrium. We estimate an empirical version of this model and see whether the observations made during the European sovereign debt crisis are in line with such a setting. We use a panel data set of 25 countries with annual observations between 1999 and 2011. The model comprises two equations: The first one tries to explain sovereign debt ratings which serve as a proxy for the probability of default of a country. The empirical strategy follows largely suggestions in the literature. Ratings depend on a vector of macroeconomic fundamental variables among which one finds the effectively paid interest rate upon government debt. The unexplained component or error term in this regression, by assumption, captures opinions of rating agencies which are not reflected by fundamentals. The second equation tries to explain the interaction between actual credit spreads on 10 year government bonds and the corresponding rating. In the latter equation we find a non-linear connection between the two variables whereas in the first one we do not. This finding supports the notion of multiple equilibria. We then use the empirically estimated model to draw conclusions with respect to the European sovereign debt crisis. It appears that countries with bad fundamentals but still in stable territory could easily be pushed beyond the unstable equilibrium, either by shocks on macroeconomic fundamentals or by rating shocks alone. Downgrading a country by a few notches might trigger a downward spiral of ever higher interest rates and worse ratings that eventually leads to default. Even more worrying is the fact that any downgrade below the A section of the rating scale might trigger events that justify the initial downgrade and are in that sense self fulfilling. The results have to be taken with care, however, since regression estimations do not necessarily prove causality but might only show a descriptive scenario based on correlations. Given previous results in the literature which show that rating changes Granger-cause higher interest rates, the findings provided in this paper are still quite startling. Even small effects might be sufficient, in specific situations, to push a country on a different path with respect to debt sustainability.

The second chapter, entitled “The role of corporate debt, risk and heterogeneity in the macroeconomy” develops a dynamic general equilibrium model with heterogeneous firms that are owned by a representative household. Firms are hit by idiosyncratic productivity shocks and finance themselves through equity and debt. A firm has to file for bankruptcy if its net worth becomes negative. Interest rates on external debt contain risk premia which reflect the risk of costly defaults. Firms thus face a trade off between distributing dividends and taking up new loans at potentially higher interest rates. A general equilibrium is reached through wage adjustment. Firms will be created as long as the return on stocks is larger than the return on the savings account. If so, the demand for labour increases as more firms are established which increases the demand for labour and thus rises wages. This reduces the profits of all firms and dividends fall. Consequently, the expected return from founding new firms declines and, eventually, an equilibrium establishes. The stationary solution of this model is solved numerically using value function iteration. The model is used to analyse how dividend policies and debt ratios vary with firm size. Numerical experiments also show how permanent shocks on the volatility of the idiosyncratic shock or the aggregate productivity affect the firm size distribution, aggregate output and wages.

The third chapter is entitled “Can relative consumption in incomplete markets explain the wealth distribution?”. Relative consumption behaviour, also known as external habits or ‘keeping up with the Joneses’ characterizes the well established observation that individuals compare themselves, their living standards and thereby, their level of consumption with other individuals around them. Utility functions reflecting this behaviour are embedded in an otherwise standard heterogeneous agent general equilibrium model with incomplete markets. The paper arrives at two conclusions. First, relative consumption behaviour leads to a more equal society under plausible assumptions. This result is surprising and works against previous attempts to explain the observed inequality in wealth holdings with these types of models. So far, factors like heterogeneous ability or heterogeneous discount rates already could hardly match the data. From here, it follows that these factors have to be even more pronounced or entirely new arguments have to be found. Second, welfare optimizing tax policies turn out to be in contrast to previous findings. A social planner could raise utilitarian welfare by implementing a progressive and thus redistributive tax either on total or even on capital income only. In a standard setting, that same tax policy reduces welfare because of the well-known disincentive effects on labour supply and capital accumulation.

Chapter 1

Rating agencies, self-fulfilling prophecy and multiple equilibria? An empirical model of the European sovereign debt crisis 2009 - 2011[1]

We explore whether experiences during Europe's sovereign debt crisis support the notion that governments faced scenarios of self-fulfilling prophecy and multiple equilibria. To this end, we provide estimates of the relation between macroeconomic variables including effective interest rates and sovereign debt ratings, and estimates of how ratings bear on current interest rates. We detect non-linearities in the latter relationship which is strong enough to generate multiple equilibria if current interest rates pass into effective interest rates. The good equilibrium is stable, ratings are excellent and interest rates are low. A second, unstable equilibrium marks a threshold beyond which the country falls into an insolvency trap from which it may only escape by exogenous intervention. Coefficient estimates suggest that countries should stay well within the A section of the rating scale in order to remain reasonably safe from being driven into eventual default.

Keywords: eurozone, crisis, sovereign debt, credit spreads, bond yields, rating agencies, multiple equilibria, self-fulfilling prophecy

JEL: F3, G24, H6

1 Introduction

Internet blogs are alive with conjectures of multiple equilibria and self-fulfilling prophecy as key characteristics of the European sovereign debt crisis, and with discussions of its

[1]This chapter is joint work with Manfred Gärtner, University of St. Gallen. It is an extension of our working paper Gärtner & Griesbach (2012)

implications.[2] *Academic journals* feature an impressive list of refined models that may generate multiple equilibria. Interest in this topic existed well before the Great Recession, as the experience of 2007-2009 is generally referred to, but intensified while the twin crises gained traction.[3] *Policy discussions* offer compelling advice as to what recipes could work in situations where good and bad equilibria coexist side by side.[4] By contrast, little, if any, direct empirical evidence appears to have been put forward in support of the actual existence of multiple equilibria in Europe's current turmoil.[5] This paper offers some evidence to this effect.

We estimate a modification of the debt crisis model suggested by Calvo (1988). In this theoretical contribution the interplay between interest rates and probabilities of default may generate multiple equilibria. Following suggestions in the literature, we estimate the influence of macroeconomic fundamentals on the probability of default measured by sovereign debt ratings. Macroeconomic fundamentals include among others the effective interest rate on government debt. In a second step, we estimate the effect of the probability of default, measured by ratings, on current government bond yields and find a non-linear relationship. Combining both results, we find that the estimates indeed support the existence of multiple equilibria. We identify thresholds on the rating scale below which downward spirals are likely to set off. Unjustified rating adjustments could potentially contribute to such a scenario and, as a consequence, would turn out to be self-fulfilling. In other words, ex ante unjustified rating changes could ex post be justified. The results have to be taken with a pinch of salt, however, since just like all our predecessors, our estimates might suffer from a potential omitted variable bias or measurement error. A series of robustness checks try to reduce this possibility as much as possible.

The next section surveys related work on multiple equilibria and self-fulfilling prophecy in the context of government debt. Section 3 describes the already mentioned Calvo (1988) model which provides the basis for our empirical analysis. Estimation results are presented in Section 4. Section 5 presents some refinements and discusses caveats of the empirical model. This follows an analysis of insolvency thresholds beyond which default

[2]For pertinent contributions see Krugman (2011), De Grauwe & Ji (2012*a*) or The Economist (2011).

[3]A classic is Krugman (1996). More directly on debt crises are Calvo (1988), a classic as well, Cole & Kehoe (2000), and, on the European debt crisis, De Grauwe (2011) and Gros (2011). Hughes Hallett & Martinez Oliva (2011) show how current account and portfolio imbalances may generate multiple equilibria.

[4]See, for example Gerlach (2010) and De Grauwe (2011)

[5]Two recent econometric papers bear indirectly on the issues of multiple equilibria and self-fulfilling prophecy in the current sovereign debt crisis. De Grauwe & Ji (2012*b*) find evidence "that a significant part of the surge in the spreads of the PIGS [...] countries in the Eurozone during 2010-11 was disconnected from underlying increases in the debt to GDP ratios, and was the result of negative market sentiments that became very strong since the end of 2010." This result is shown to apply to a wider set of fundamentals in De Grauwe & Ji (2012*b*). Similarly, von Hagen et al. (2011) report that "markets penalise fiscal imbalances much more strongly after the Lehman default in September 2008 than before." Blanchard (2011) considers the 'facts' so convincing that he states: "post the 2008-09 crisis, the world economy is pregnant with multiple equilibria" as the first of the hard truths he learned from 2011.

appears unavoidable without outside help. Section 6 concludes.

2 Related work

The field of debt crises due to multiple equilibria and self-fulfilling prophecies took off with the influential work of Calvo (1988).[6] Even though models with multiple equilibria circulated much earlier, they were mostly considered to be theoretical artefacts of possibly misspecified models. Calvo kick-started the idea that multiple equilibria were a phenomenon worth analysing instead of discarding it as many others had done before him. His simple two-period model of pricing government debt highlights the fact that for specific parameter values multiple equilibria occur. Self-fulfilling expectations can lead to any one of them.

Alesina et al. (1990) jump on that idea and empirically analyse the debt structure of Italy during the 1980s discussing the origins of and the remedies for self-fulfilling debt crises.

Cole & Kehoe (2000) extend Calvo's model by embedding it into a Dynamic Stochastic General Equilibrium framework. They show that specific constellations of a country's fundamental values such as the debt level, maturity structure and private capital stock can move it into a so-called crisis zone. Here, the probability of default is no longer a function of fundamental values but is determined by the beliefs of market participants. Cole & Kehoe also suggest certain policy actions for countries to cope with being in the crisis zone, such as reducing debt or increasing average maturity. However, they point out that the best strategy would be to never get into the crisis zone in the first place, i.e. to keep debt levels below a certain threshold. The authors also refer to their own, earlier work, Cole & Kehoe (1996), which provides empirical support for their model. Using a calibrated model they confirm that Mexico, in its 1994 crisis, was probably in such a crisis zone.

A similar argument is found in Masson (1999*b*), namely, that market sentiment and self-fulfilling expectations - not fundamental variables alone - explain the spread of the crisis in Mexico and East Asia in 1997. Masson (1999*a*) further develops the idea that crisis *contagion* can be more easily explained in an environment of multiple equilibria and self-fulfilling expectations.

Another early contribution in the area of sovereign debt crises comes from to Alesina et al. (1992). Using regression analysis on a panel data set of 15 OECD countries they show that a selection of fundamental variables influences the perceived default probability. As a proxy for the latter variable the authors use either the ratio of the public interest rate to the private interest rate or the difference between those rates. The fundamentals chosen are the public debt ratio, the change in that ratio, industrial production and the

[6]See also Romer (2011) who gives an excellent, brief introduction into the topic. A related field in the literature, worth mentioning, is that of self-fulfilling currency crisis, spearheaded by Krugman (1996).

average maturity of public debt. While their results do not give an entirely clear picture, and raise some questions,[7] they emphasize the positive influence of the debt level on perceived default risk. They also argue that high debt levels lead to the possibility of multiple equilibria in the sense of a self-fulfilling confidence crisis.

A purely theoretical contribution on multiple equilibria in sovereign debt pricing is due to Detragiache (1996). His model emphasizes the necessity of a market that consists of many small investors in order to explain situations with multiple rational expectations equilibria. Pessimistic expectations of creditworthiness may then trigger a liquidity crisis.

Among the first to put rating agencies into the game, in the sense that ratings might have an influence on outcomes if multiple sunspot equilibria exist, were Kaminsky & Schmukler (2002). In a panel regression they show that sovereign debt ratings do not only affect the bond market but also spill over into the stock market. This effect is stronger during crises, which could be explained by the presence of multiple equilibria. As a consequence they claim that rating agencies contribute to the instability in emerging financial markets. Carlson & Hale (2005) argue that if rating agencies are present, multiple equilibria emerge in a market in which otherwise only one equilibrium would exist. The purely theoretical paper is an application of global game theory and features heterogeneous investors. Boot et al. (2006) arrive at the opposite conclusion: ratings serve as a coordination mechanism in situations where multiple equilibria loom. Using a rational-herd argument,[8] they show that if enough agents base their investment decisions on ratings, others rationally follow. Since ratings have economic consequences, they emphasize that the role of rating agencies is probably far greater than that of the self-proclaimed messenger.

After the outbreak of the recent financial and sovereign debt crises many new papers unsurprisingly appeared in this field. Besides our own work in Gärtner et al. (2011), Arezki et al. (2011) also find a significant effect of sovereign rating news on credit markets during 2007-2010. This is in line with results provided by Kiff et al. (2012), who find that ratings affect the cost of funding of sovereign issuers and are, therefore, a threat to stability in sovereign bond markets. Multiple equilibria and self-fulfilling prophecies are also addressed in De Grauwe (2011), De Grauwe & Ji (2012*a*), De Grauwe & Ji (2012*b*) and Corsetti & Dedola (2011).

3 Theory

The backbone for our empirical analysis is provided by Romer's (2011) structural adaption of Calvo's (1988) optimizing model of sovereign debt crises. These models look at the interaction between interest rates on government bonds and expected probabilities

[7]There is a critical discussion appended to the paper with comments by Maurice Obstfeld, Sergio Rebelo, Martin Hellwig, Hans-Werner Sinn, etc. who, among other things, criticize the construction of the dependent variable, the risk premium.

[8]See Chamley (2003) on an introduction to rational herds.

of sovereign default, where, ceteris paribus, the interest rate is assumed to bear on the likelihood of default, and the likelihood of default affects the interest rate; the models analyse the rational-expectations equilibria that may arise in such a setting.

It should facilitate the interpretation of the empirical work to follow below if we take a brief look at the Romer model and its graphical representation. The model consists of two equilibrium conditions. The first one focuses on the fact that, in each period, the government decides whether to service its debt or to go into default. This decision depends on its ability to pay, i.e. the difference between the tax revenue, and the required interest payments, $i \times D$, where i is the interest rate being paid on debt D. For given tax revenue and debt, the higher the interest rate, the higher the probability of default. Thus

$$p = F(i, D) \qquad\qquad F_i, F_D > 0 \tag{1}$$

Figure 1 displays this relationship as a Z-shaped line. Up to an interest rate i_1, servicing the debt is painless and there is no relevant risk of default. Beyond i_2 servicing the debt would drain money from so many other critical policy areas that it would be political suicide. In this situation, going into default remains the only feasible option. In the interval between these interest rates the default risk increases monotonically.

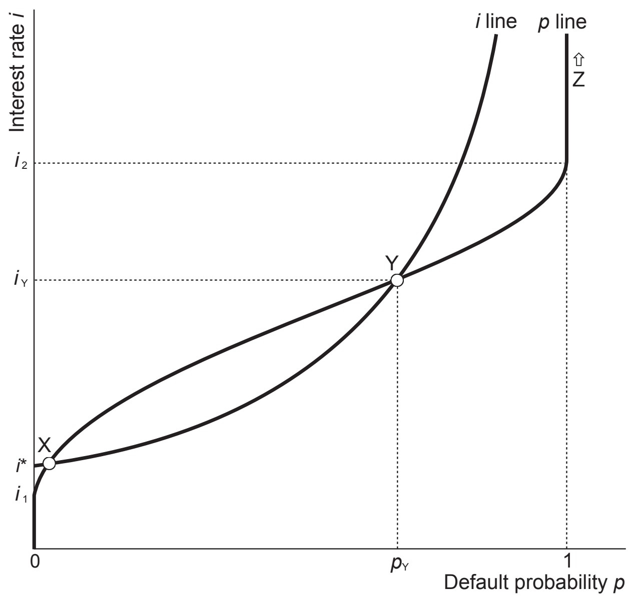


Figure 1: The Romer model of sovereign debt crises.

The second condition renders investors indifferent between some exogenous risk-free interest rate (or rate of return) i^* and the interest rate i for a government bond with an attached non-zero default probability of p. If default is a one-off event, creditors are risk neutral, and the government only issues one-period bonds, this equilibrium condition reads $(1 - p)(1 + i) = 1 + i^*$, or

$$i = \frac{i^* + p}{1 - p} \tag{2}$$

In the diagram shown in Figure 1 that features the default probability on the horizontal axis and interest rates on the vertical axis, equation (2) is displayed as a hyperbola-shaped curve, which intersects the ordinate at the risk-free interest rate.

When curves are positioned as in Figure 1, the model generates three equilibria. The first one is the point of intersection between the two lines in the lower left part of the diagram (point X). In this good equilibrium the interest rate is low and the government is likely to honour its commitments. The second equilibrium, at point Y, implies a substantial spread between the interest rate the government pays and the risk-free alternative, reflecting a significant risk of default. In the third equilibrium, interest rates rise so high that default becomes certain. So nobody purchases the government's debt and the government is forced to default. This 'equilibrium' cannot be identified by a point since the market for this country's debt titles has broken down and no interest rate is determined.

Equations (1) and (2) describe an equilibrium model but do not really say anything about dynamics. As spelled out by Romer (2011, pp. 637 ff.), however, under plausible assumptions such as permitting a lagged response by interest rates to changes in default probabilities,[9] the first and the third equilibrium are stable. The one in the middle, point Y, however, is unstable and functions as a threshold. Once this threshold has been passed, default becomes very much inevitable.

Of course, three equilibria only obtain when the relative curve positions are as shown in Figure 1. If we move the i curve far enough up, the good equilibrium disappears and, without outside intervention, default cannot be avoided. If we move the i curve sufficiently far down, we are left with the good equilibrium only, which is then globally stable.

4 Empirical model

We now ask whether the empirical data support an interpretation of recent developments in financial markets in line with Romer's model. The model employed here differs from the Calvo-Romer scenario in two ways. First, we replace the *default probability* of sovereigns by sovereign bond *ratings*. This variable is easily observed and measured. It also permits us to discuss how potential misjudgements of rating agencies may affect the dynamics of a sovereign debt crisis. Second, acknowledging the evidence of how expectations may be formed in financial markets during normal times,[10] and acknowledging the often expressed view that markets in the aftermath of the financial crisis often appeared to be

[9] This would follow from discarding the notion of rational expectations and replacing it by some adaptive scheme.

[10] See Haruvy et al. (2007, p. 1901) who conclude: "We find that individuals' beliefs about prices are adaptive".

driven by panic and fear, or even schizophrenia, rather than rationality,[11] we look beyond rational expectations equilibria to permit bandwagon and herd behaviour to allow for institutional influences, and thus warrant a richer set of dynamics.

Our model comprises the same two propositions as the Romer model. First, the probability of default as measured by sovereign bond ratings is affected by the ability to service the outstanding debt in the future. This ability is ultimately determined by a vector of currently observable macroeconomic variables $\mathbf{N}$ that have an influence on the future dynamics of debt. Most importantly and at the centre of our discussion is the effective interest rate a country pays on its current amount of debt. In contrast to the Romer model, in which the current and the effective interest rate are equivalent since debt has a maturity of one period, we use the lagged one year average interest rate on government bonds, $\bar{\imath}_{-1}$, as a proxy. The remaining variables in $\mathbf{N}$ are fundamentals that reflect a government's budget and debt policies. There is no clear answer as to which variables ultimately determine the rating of a country.[12] We use a set of variables that were previously identified in pertinent research as being the main driver of sovereign debt ratings:[13] growth trend, income per capita, budget surplus, primary surplus, debt ratio and inflation. Section 4.1 provides the details on these variables. The probability of default measured by sovereign bond ratings, also comprises subjective appraisals by the rating agencies that are not measurable with macroeconomic fundamentals. A good example might be the political willingness to pay. These unobservables naturally end up in the error term ϵ. Hence, we are left with the following equation:

$$r = \Psi(\mathbf{N}, \epsilon) \qquad \text{with } \bar{\imath}_{-1} \in \mathbf{N} \tag{3}$$

[11]These include both successful practitioners and eminent academics. Soros (2012), a legend in investment circles, quips: "I am not well qualified to criticize the theory of rational expectations and the efficient market hypothesis because as a market participant I considered them so unrealistic that I never bothered to study them." Two of the IMF chief economist Blanchard (2011) four hard truths he learned from 2011 directly bear on this. His number 3 is: "financial investors are schizophrenic about fiscal consolidation and growth". And the fourth reads: "Perception moulds reality". See also Arezki et al. (2011).

[12]Rating agencies remain secretive about their true procedures. In their sovereign ratings methodology Fitch claims to use 148 *groups* of variables for their rating assessment. However, they do not provide a precise answer as to how they aggregate all that information into a single variable. A procedure based on statistical methods is highly unlikely given the limited data set of currently 112 rated countries and the obviously limited time frame. See Fitch, Inc. (2011).

[13]See Cantor & Packer (1996), Ferri et al. (1999), Mellios & Paget-Blanc (2006) and El-Shagi (2010) who all report regressions on sovereign debt ratings explained by fundamentals. We also consider all variables which affect the well-known debt dynamics equation; see Escolano (2010) for an introduction to this topic. See also the discussion on the selection of variables in our previous research, Gärtner et al. (2011), in which we use the same set of variables as in this study. Some variables used in other studies are omitted for plausible reasons. For example, the default history does not play a role in our sample, since none of the selected OECD countries has declared default after World War II. Other variables mentioned in the literature have been tested but turned out to be either non significant or led to multi collinearity due to high correlation with already included variables. Following the literature, we also abstain from using forward looking variables such as expected growth rates. Forecasts are unreliable, have a large variance and are only available for very short time horizons. Since rating agencies emphasize that they do not build their judgements on short run business cycle fluctuations, it is plausible to rather use trends of variables that are very volatile, than one year forecasts.

Second, current interest rates i on government debt comprise risk premia and are consequently affected by the expected probability of default as signalled by the sovereign bond rating r plus, potentially, a vector of macroeconomic variables which market participants may assume to affect the risk-free interest rate. The latter would, thus, affect the position of the interest rate line in Figure 1. However, we follow an even simpler approach by using the German interest rate as a direct representative of the risk-free rate and thereby make other macroeconomic variables redundant. We leave room for other functional forms, since the hyperbola derived as equation (2) only obtains under a set of restrictive assumptions. We get

$$i - i_D = \Omega(r) \qquad \Omega_r > 0 \tag{4}$$

where $i - i_D$ denotes the current credit spread of the respective country versus Germany. We abstain from adjusting for forward exchange rates in the benchmark setting, as we use foreign currency ratings for our analysis which include potential exchange rate risk.[14]

4.1 Data description

Our empirical analysis uses annual data for 25 OECD countries for the period 1999 to 2011.[15] Nine OECD members were omitted because no data was available or because they became members after 1999. We chose this specific period because sovereign ratings for the observed countries are not always available before then, and in order to avoid the structural break due to the introduction of the Euro in 11 countries of our sample. As explained in the previous section, we include the following variables in our regressions. Descriptive statistics are shown in Table 1:[16]

- *Rating*: Three major agencies provide sovereign ratings: Moody's, Fitch Ratings and Standard & Poor's. We use end-of-year, long-term foreign currency sovereign debt ratings of Fitch Ratings.[17] We convert the ratings into an equidistant numerical scale running from 1 for D to 21 for AAA following other studies on rating agencies, such as Afonso et al. (2007). As a robustness check we also use long-term local currency ratings of Fitch Ratings to take expected currency fluctuations into account.

[14] As a robustness check, we provide estimations with local currency ratings and credit spreads adjusted for forward exchange rates in the Appendix (see 7.4.2).

[15] Our sample includes Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Poland, Portugal, South Korea, Spain, Sweden, Switzerland, United Kingdom, and the USA.

[16] Our macroeconomic data for 2011 are mostly estimates provided by the OECD Economic Outlook or the IMF World Economic Outlook database.

[17] Since the data needed for our analysis were not available for all three agencies we settled for Fitch as a representative. Given the high correlation between the ratings of the three big rating agencies, we do not expect this choice to bear on our main results. Note that, for example Gaillard (2011) report pairwise correlation coefficients between the three rating agencies above 0.97 during 2000 until 2010.

- *GDP growth*: Data on real GDP growth is from the OECD Economic Outlook No. 90 Annex Table 1. Following the literature, we use three year averages which reflects the agencies' approach to take out the business cycle effect when deciding on a sovereign rating.[18]

- *GDP per capita*: Real GDP per capita measured in thousand current international dollars is from the IMF World Economic Outlook database.

- *Budget surplus*: This variable measures general government financial balances as a percentage of nominal GDP and includes one-off factors such as sales of mobile phone licenses. The source is Annex Table 27 of the OECD Economic Outlook No. 90.

- *Primary surplus*: This variable measures the general government underlying primary balance as a percentage of potential GDP. It equals government surplus less net interest payments and is adjusted for one-off factors. The source is Annex Table 30 of the OECD Economic Outlook No. 90.

- *Debt ratio*: General government gross debt as a percentage of nominal GDP is taken from Annex Table 32 of the OECD Economic Outlook No. 90.

- *Inflation*: Consumer price inflation is taken from Annex Table 18 of the OECD Economic Outlook No. 90.

- *Bond yield*: This is the annual average of monthly 10-year generic government bond yields as provided by Bloomberg.

- *Credit spread*: The credit spread is calculated as the difference between the December value of the monthly 10-year generic government bond yield of a country and that of Germany.

- *Expected currency gains*: Based on one year forward exchange rates provided by Bloomberg we calculate the expected currency gains in percent with respect to the Euro, i.e. the currency of Germany. To match the date of credit spreads, we use December values, too.

4.2 Estimating the rating equation

We start with the rating equation, which attempts to quantify how interest rates on government bonds and other economic variables affect the probability of default as measured by the credit rating of the country. Regression results are shown in Table 2.

Column 1 reports the key result which may be interpreted as a linearisation of equation (3) around its mean. The results are in line with the ones reported in Gärtner et al.

[18]See, for example, Afonso et al. (2007).

Table 1: Descriptive statistics.

	Mean	Median	Std. Dev.	Maximum	Minimum
Foreign currency rating (r)	2.7	1.0	2.5	18.0	1.0
Local currency rating (r_L)	2.4	1.0	2.2	18.0	1.0
GDP growth	2.2	2.3	1.9	9.0	-4.3
GDP per capita	30.9	30.5	8.1	53.4	11.1
Budget surplus	-1.7	-2.0	5.3	19.1	-31.3
Primary surplus	-0.4	-0.1	3.0	6.9	-8.2
Debt ratio	68.2	61.1	34.4	211.7	13.7
Inflation	2.3	2.3	1.5	9.8	-1.7
Bond yield ($\bar{i}$)	4.6	4.4	1.7	19.1	1.0
Credit spread ($i - i_D$)	0.8	0.3	2.5	33.1	-3.6
Expected currency gains (ϵ_D)	-0.5	0	1.6	4	-9.5

Notes: Unit root tests can be found in section 7.2 of the Appendix.

Table 2: Regressions explaining sovereign debt ratings.

	(1)	(2)	(3)
Constant	1.214	1.886	1.723
	(2.408)	(2.376)	(2.139)
GDP growth	−0.049	−0.050	−0.044
	(0.105)	(0.105)	(0.111)
GDP per capita	−0.118***	−0.113***	−0.117***
	(0.037)	(0.039)	(0.033)
Budget surplus	−0.013	−0.021	−0.015
	(0.053)	(0.057)	(0.051)
Debt ratio	0.022***	0.020**	0.021***
	(0.008)	(0.009)	(0.007)
Primary surplus	−0.141**	−0.126	−0.135**
	(0.070)	(0.080)	(0.068)
Inflation	0.178*	0.191*	0.187
	(0.105)	(0.113)	(0.116)
$\bar{i}_{-1}$	0.693***	1.013**	0.541*
	(0.219)	(0.476)	(0.288)
$\log(\bar{i}_{-1})$		−1.469	
		(1.772)	
$\bar{i}^3_{-1}$			0.001
			(0.002)
R^2	0.608	0.611	0.610
adjusted R^2	0.598	0.600	0.599
Durbin-Watson	0.714	0.728	0.734
F	14.079	16.738	16.492
p	0.000	0.000	0.000
Observations	291	291	291

Notes: Pooled OLS regressions. The dependent variable is *foreign currency rating* r. $\bar{i}$ denotes government bond yields. Heteroscedasticity and autocorrelation robust standard errors in parentheses as documented in Zeileis (2004). *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

(2011) which does not come as a surprise, since the only difference is an additional year of country observations. Sovereign bond ratings are found to depend on the macroeconomic indicators typically used in pertinent empirical research as explained above. These indicators explain roughly 60 percent of the variance of sovereign bond ratings in our panel. All estimated coefficients possess the expected signs, though not all are significantly different from zero. Ratings are found to improve with higher income growth and income levels, or with better overall and primary budget situations. Ratings deteriorate when the debt ratio, inflation or government bond yields go up.

A crucial question from the perspective of the model suggested in section 3 is whether the regression line in an r-i diagram is linear or not. We did not find any evidence in support of non-linearity. Applying the test proposed in Davies (1987), the null hypothesis of a constant slope coefficient could not be rejected. Similarly, non-linear functional relationships did not improve the fit of the regression equation. Columns 2 and 3 are representative of these efforts, showing that convex or concave shapes of the effect of bond yields on ratings lower confidence and fit levels.[19] This suggests that the ratings curve of our model may be considered linear within the range covered by our panel data.

We observe a considerable amount of autocorrelation which might either be attributed to misspecification or, simply, to autocorrelation in the error terms, which in our case reflect qualitative judgements of rating agencies by assumption. Both, of course, is possible. As rating agencies do not reveal their rating procedures, every attempt to unveil them will have to deal with the possibility of misspecification.[20] However, theory tells us that, from a fundamental point of view, our model is correctly specified. The error term in our regression, thus, captures mostly qualitative opinions by rating agencies that are not included in our set of fundamental variables. These opinions might, however, be correlated with the fundamentals and thereby bias our results. To partially cope with this problem, we provide a fair amount of robustness checks in section 7.3 of the Appendix which are mostly consistent with the findings presented here.[21] Hence, we stick to the assumption of autocorrelated error terms per se. This assumption could be justified by arguing that rating agencies, at least in the more distant past, only inertially changed ratings even though fundamentals would imply a different picture. It is well known, that autocorrelation in the error terms leads to consistent but non efficient estimation results. Therefore, heteroscedasticity and autocorrelation robust standard errors as suggested by Newey & West (1987) are shown in Table 2. When comparing these standard errors with

[19] In section 7.3.3 of the Appendix we also run a generalized additive model (GAM) with penalized regression splines as smoothing terms. The optimal functional form with respect to the interest rate is again a straight line.

[20] Misspecification could occur in different ways in this setting. Important explanatory variables that are correlated with the given regressors could be missing. This would lead to an omitted variable bias. Secondly, explanatory variables could have non-linear effects in this equation. Also, it is possible, that ratings might just not measure the probability of default correctly, leading to a measurement error.

[21] In section 7.3 we show fixed effects estimations, generalized additive models and a cross sectional analysis all of which essentially do not invalidate the results shown here.

the normal standard errors (see section 7.3.1 in the Appendix) it is obvious that changes are minimal. We take that as a signal that the observed amount of autocorrelation is not such a big threat to our results.

4.3 Estimating the interest rate equation

We now turn to the effect of sovereign bond ratings on (current) interest rate spreads. Baseline results are given in Table 3.[22]

Table 3: The effect of ratings on sovereign bond yields (I)

	(1)	(2) a	(2) b	(3)	(4)
Constant	−0.961**	−0.197	−22.888***	−0.450***	0.156
	(0.482)	(0.193)	(0.580)	(0.162)	(0.129)
r	0.657***	0.301***	3.119***	0.433***	0.032
	(0.213)	(0.062)	(0.034)	(0.070)	(0.057)
$\exp(r)$				0.000***	
				(0.000)	
r^3					0.006***
					(0.000)
R^2	0.420	0.239	0.997	0.735	0.799
adjusted R^2	0.418	0.236	0.996	0.733	0.798
Durbin-Watson	1.132	0.601	1.198	0.854	0.784
F	9.503	23.254	8326.864	60246.438	611.967
p	0.002	0.000	0.000	0.000	0.000
Observations	291	284	7	291	291

Notes: Pooled OLS regressions. The dependent variable is *credit spread* $i - i_D$. r denotes *foreign currency rating*. Heteroscedasticity and autocorrelation robust standard errors in parentheses as documented in Zeileis (2004). *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table. The combined R^2 from regressions (2a) and (2b) is 0.805 and the adjusted R^2 is 0.805, too.

The regression reported in the first column proposes a simple linear relationship between the interest rate and the sovereign debt rating and serves as a reference point. It suggests that an AAA rated country, which translates into a numerical value of 1 for the rating variable, may expect an interest rate spread versus Germany of $-0.961 + 0.657 = -0.304$. Any downgrade by one notch raises this country's interest rate by 65.7 basis points. This equation explains 42 percent of the variance in the credit spreads in our sample.

As suggested by the non-linear functional form of the interest rate equation in the Romer model, shown as equation (2) above, we next explore whether the regression coefficient is really constant over the entire range of rating observations. To this end we test the null hypothesis of a constant slope coefficient, as provided in the first column, against the alternative hypothesis of a break in the regression line. Applying the test proposed in Davies (1987), the null hypothesis of no break was rejected, and the break

[22]Note that the data allows to estimate this equation also at higher frequencies, for example monthly or even daily. However, since we later discuss interactions between the rating and the interest rate equation, we stick with annual data time series.

point was found to lie between a BBB+ and a BBB rating.[23] Regression estimates for the resulting two segments are shown as regressions 2a and 2b in Table 3.

The differences between the two segments are striking. The slope coefficients differ by a ratio of ten to one. While, on average, a rating downgrade by one notch raises interest rates by 0.3 percentage points when ratings are in the range between AAA and A-, which comprises seven categories, a downgrade by one step raises the interest rate by 3.12 percent once the rating has fallen into the B segment or below. Both coefficients are highly significant, though. Fit levels differ substantially, with adjusted coefficients of determination of 0.239 and 0.996, respectively. These refer to quite different sample sizes, however, which comprise 284 observations in the first segment, and only 7 observations in the second. The empirical relevance of permitting non-linearity in the relationship between interest rate spreads and ratings is underscored by the increase of the adjusted coefficient of determination for the entire sample, which is 0.418 for the linear model and 0.805 for the combined parts of the segmented regression.

Given that the null hypothesis of a constant slope coefficient was rejected, we tried different functional forms to represent the impact of ratings on spreads. Almost any function that permits a convex shape of the i curve, as proposed in the Romer diagram, generates a significantly improved fit of the estimation equation. For example, using an exponential function, as reported in regression 3 in Table 3, boosts the adjusted coefficient of determination to 0.733. Similar results are found with reciprocal specifications, or when employing polynomials.

After some experimentation, it turns out that a third order polynomial provides the best results from a goodness-of-fit perspective.[24] Matters are simplified by the fact that the first-order and second-order terms are not significant at conventional levels. Thus, we are left with the simple regression reported in the last column of Table 3.[25] This shows that raising ratings to their third power rather than employing them linearly virtually doubles the adjusted coefficient of determination, from 0.418 to 0.798. The implied non-linearity is substantial: depriving a country of its AAA status, downgrading it by one notch, raises the interest rate by 0.050 percentage points only. Doing the same thing to a BBB+ country raises the interest rate by 1.490 percentage points. But if a country with a CCC+ rating slides down one more step, the interest rate its creditors require increases by a whopping 5.864 percentage points.

As in the previous section we observe a significant amount of autocorrelation in the residuals. Again we hypothesize that this is mostly due to inertia in the ratings themselves. Attributed to this fact, Table 3 also displays heteroscedasticity and auto-

[23] For the employed methodology see Davies (1987) and Muggeo (2003). Regressions were run using the R package provided in Muggeo (2008) adjusted for heteroscedastity and serial correlation according to Newey & West (1987) and Zeileis (2004).

[24] As before, we also estimate a generalized additive model (GAM) in section 7.4.1 in the Appendix. It turns out that three degrees of freedom are effectively used.

[25] Regression 4 in Table 3 still shows the non significant linear term to demonstrate non-linearity once more.

correlation robust standard errors as suggested by Newey & West (1987). To the best of our knowledge we could not find any indication that our model is misspecified. Prove of our effort is the bulk of robustness checks provided in section 7.4 of the Appendix.[26]

As mentioned above, we also investigate the potential influence of exchange rate risk by looking into local currency ratings and credit spreads adjusted for forward exchange rates in section 7.4.2 of the Appendix. Eventually we also limit our country sample to the Euro countries. Still, our results are rather stable.

4.4 A note on causality

The regressions provided in the previous section cannot show a causal relationship from ratings on current credit spreads or interest rates. Both could be influenced by an unobservable underlying probability of default. Since it is difficult to observe this variable, it is hard to identify whether a rating change on its own has an impact. If this was not the case, ratings might only be a proxy after all, potentially, with a measurement error attached in certain cases. Markets might ignore any deviation of ratings from the fundamental underlying probability of default and rating agencies would not have any impact on the sovereign debt market. As highlighted in the following paragraphs, however, there is a considerable amount of evidence in the literature showing that ratings do have an effect on interest rates. Combined with the multiple equilibria setting described above this opens the possibility of an even greater active role of sovereign debt ratings than previously thought. Initially small effects might be enough to push a country beyond the threshold point Y in Figure 1 from where default becomes inevitable.

Kaminsky & Schmukler (2002) provide evidence that changes in sovereign debt ratings affect the prices of bonds. This study focuses on emerging markets in the last decade of the twentieth century. Reisen & von Maltzan (1999) report similar results but focus on the East Asian crisis. Arezki et al. (2011) show that there is an impact of rating news on credit markets in Europe during the time span 2006-2011. Afonso et al. (2012) support this finding while working with a a different data set which lasts from 1995 until 2010. Most of these studies use daily data in a VAR framework and apply Granger-causality tests. Similar results are reported in Gärtner et al. (2011).[27]

Looking at corporate ratings instead of sovereign ratings Kliger & Sarig (2000) study Moody's refinement of its rating system in 1982. Even though the underlying fundamental probability of default was not affected, the rating changes triggered significant price changes, leading to the conclusion that ratings do have a causal effect. A similar, exogenous rating event that has not yet been investigated statistically, is the erroneous rating downgrade of France on November 10, 2011: By mistake Standard & Poors announced

[26]Fixed effect regressions do not alter the results. Also, running the regressions on a cross section sample or taking first differences of the relevant variables to cope with the possibility of non-stationarity deliver similar findings.

[27]Several other studies emphasize the impact of rating agencies in credit markets such as Eichengreen & Mody (2000), Ferri et al. (1999), Manganelli & Wolswijk (2009) and Kiff et al. (2012).

that France was downgraded from AAA to AA+. Two hours later this announcement was withdrawn and France's top rating was reaffirmed. During these two hours, French government bond yields had surged to a five month high.[28].

Leaving statistical and econometric evidence for a causal relationship aside, one can also argue that institutional factors necessarily lead to such a relationship. The Basel II framework forces European banks to keep a minimum capital ratio of 8%. This capital ratio is calculated using a risk weighted sum of total assets to which ratings by the three big agencies have to be taken into account as long as a bank cannot provide its own rating. Secondly, the European Central Bank, but also other financial institutions are forced by law to only keep investment grade securities, i.e. BBB or better, in their books. Downgrading these securities forces fire sales which detoriate prices and increase the respective interest rates. Early warnings about this issue can be found in Kashyap & Stein (2004) who anticipate that Basel II will "exacerbate business cycle fluctuations". Other authors, such as Eichberger & Summer (2005), Pederzoli et al. (2010) and de Walque et al. (2010) also emphasized this threat.

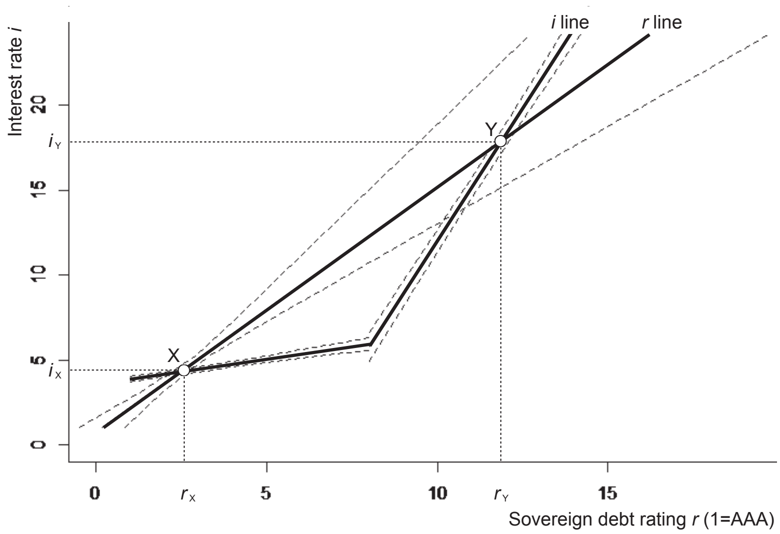
4.5 Multiple equilibria

We may now return to our key question of whether the developments observed during Europe's debt crisis support the notions of self-fulfilling prophecy and multiple equilibria. We do so on the basis of the model described and discussed in Section 3, which our estimates attempt to quantify. Figure 2 visualizes the r and i lines from our estimated model along with confidence bands. The r line in Figures 2(a) and 2(b) is derived from regression 1 in Table 2. The i line in Figure 2(a) represents the segmented regression 2a and 2b in Table 3. Figure 2(b) shows the i line representing regression 4 of Table 3 which uses a third-order polynomial term for the sovereign debt rating. In both cases the estimated credit spread equations are solved for the interest rate by adding the average German interest rate to obtain the respective i line. Similarly, the position of the r line is determined by a set of exogenous variables, which are different for each country and which change over time. To provide for a synthesized general discussion, these exogenous variables are set to their average values in our panel data set in both panels of Figure 2.

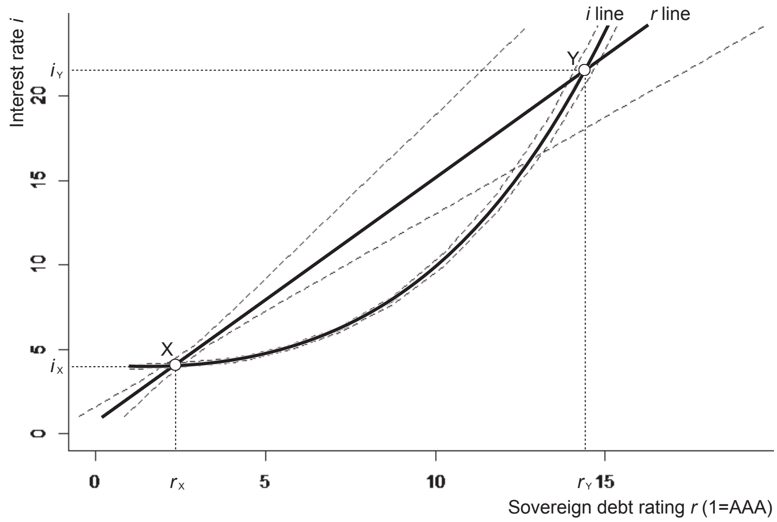
By endogenising the interest rate as it is done in Figure 2 we implicitly equate the current interest rate from the i line with the lagged average interest rate $\bar{i}_{-1}$ from the r line. As mentioned above, the latter serves as a proxy for the effective interest rate. Doing so implies that the relationship plotted in Figure 2 is rather long term.[29]

[28]See Bloomberg (2011)

[29]Endogenising other macroeconomic variables such as income, inflation, deficits or debt would require the integration of the debt crisis model into a full-scale macroeconomic model. Doing so might open a Pandora's box of issues on which economists do not agree, ranging from the magnitude of multipliers to the relevance of Ricardian equivalence. In order to sidestep this, we stick with a conservative approach here, and settle for an informal discussion of how the endogenisation of selected macroeconomic variables may affect our results. See Section 5.



(a) The empirical model with a segmented interest rate line.



(b) The empirical model with a third-order polynomial interest rate line.

Figure 2: The empirical model with different functional forms. Thin lines show 95% confidence bands.

Both empirical models feature two points of intersection between the r and i lines, and identify three equilibria. One of these is a 'degenerated equilibrium' in which a country is driven into insolvency. This corresponds to the equilibrium we labeled Z in Figure 1 above. The first and the second equilibria replicate and have the same properties as equilibrium points X and Y in Figure 1. Point X marks the unique rational ratings equilibrium. Only here does a sovereign rating generate the very interest rate that, in turn, justifies this rating. Point X is a 'good equilibrium' which is *locally stable*, i.e. stable as long as ratings remain below r_Y. Thus, Y marks a vital threshold, a point of no return. Once Y is crossed, the country drifts towards insolvency and can only be rescued by exogenous intervention.

5 Refinements and potential implications

We now move beyond the model described in Section 3 and look at some lessons suggested by our estimation results.

5.1 Refinements

The estimates provided by the regressions 5 and 6 of Table 4 derive from a more flexible interpretation of the Romer model.

Table 4: The effect of ratings on sovereign bond yields (II)

	(4)	(5)	(6)
Constant	0.156	0.144	0.128
	(0.129)	(0.126)	(0.124)
r	0.032	0.055	0.044
	(0.057)	(0.049)	(0.049)
r^3	0.006***	0.005***	0.005***
	(0.000)	(0.000)	(0.000)
Δr		0.528***	
		(0.149)	
Δr^+			0.740***
			(0.166)
Δr^-			−0.002
			(0.208)
R^2	0.799	0.814	0.819
adjusted R^2	0.798	0.812	0.816
Durbin-Watson	0.784	0.845	0.815
F	611.967	907.567	910.039
p	0.000	0.000	0.000
Observations	291	290	290

Notes: See notes for Table 3. Regression 4 is repeated for convenience. $\Delta r = r - r_{-1}$ denotes rating changes. $\Delta r^+ = \max(0, \Delta r)$ denotes downgrades, whereas $\Delta r^- = \min(0, \Delta r)$ denotes upgrades, only.

Regression 5 explores the hypothesis of whether any rating change that brings a country into the news and unsettles the financial markets, has an effect on interest rates. This effect may be independent of the actual debt rating of a country, and would only exist during the period in which the downgrade was announced. Augmenting our equations by an explanatory variable Δr does indeed generate a highly significant coefficient and the fit improves substantially. The coefficient of determination increases from 0.798 to 0.812.

It may not come as a surprise that these shock effects are not symmetric. When regression equation 6 splits Δr, which includes rating downgrades as well as upgrades, into Δr^{+}(i.e. $\Delta r > 0$) and Δr^{-} (i.e. $\Delta r < 0$), the coefficient of determination creeps up still further. The estimated coefficients differ in magnitude and are only significant when rating *down*grades are made public.

The presence of Δr^{+}, the coefficient of which carries a t-statistic of 4.46, generates some interesting and potentially disquieting dynamics. The immediate response of the interest rate to a rating downgrade is given by[30]

$$\frac{\Delta i}{\Delta r} \approx 0.044 + 0.015 r^2 + 0.740 \tag{5}$$

Now recall that the slope of the rating line represented by regression 1 in Table 2 is $\frac{1}{0.693} = 1.443$. According to equation (5), the interest rate line is steeper than this at levels of $r \geq 6.63$. This means that at sovereign debt ratings outside the lower part of the A-segment, i.e. of A- or worse, a downgrade generates an increase in the interest rate that justifies or more than justifies the initial downgrade, and may trigger a spiral of successive and eventually momentous downgrades. Only countries in the A-segment of the rating scale appear to be safe from this, at least when the shocks to which they are exposed to are only small. However, this only applies when marginal rating shocks occur. Larger shocks, and these have not been the exceptions during Europe's sovereign debt crisis, may even jeopardize countries which were in secure A territory. We may illustrate this by looking at the impulse responses implied by the system of regression equations 6 from Table 4 and 1 from Table 2 for the rating and interest line to shocks of various kinds and magnitudes. This provides us with *insolvency thresholds* that identify the size of a rating downgrade required to destabilize the public finances of countries with a given sovereign debt rating. Figure 3 summarizes the results.

Figure 3(a) looks at temporary rating shocks, as they would be captured by the error term of our regression equation. The vertical line a BBB- (= 10) rating, upwards indicates that the equilibrium is inherently unstable and that the smallest of shocks suffices to trigger an accelerating debt crisis. The outward-sloping segment shows how rating shocks to a country located in the range BBB (= 9) and better need to be increasingly larger to destabilize the country. For example, a country with a A (= 6) rating would have to be subjected to an arbitrary downgrade of five notches to a BB+ (= 11) rating in order to be pushed beyond the insolvency threshold in the long run.

When rating shocks last, however, as has apparently been the case for the euro-zone's PIGS members, much smaller unsubstantiated rating changes may play havoc with government bond markets and suffice to run initially healthy countries into trouble,

[30] This is an approximation, of course, since we are dealing in discrete time.

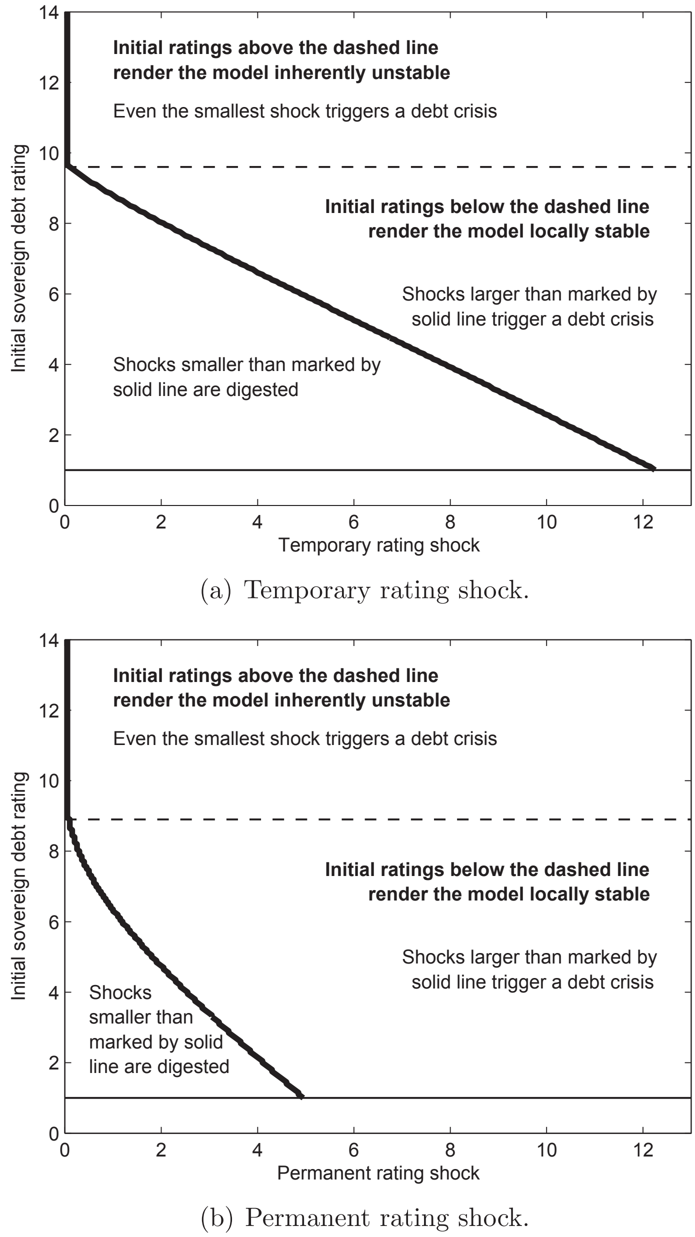


Figure 3: Insolvency thresholds under temporary and permanent rating shocks.

as shown in Figure 3(b). In this scenario, an arbitrary, yet persistent, downgrade by two notches would trigger a downward spiral in a country with an A+ (= 5) rating. Rising interest rates would call for further downgrades, which would appear to justify the initial downgrade as an apparently good forecast.

The thresholds depicted in Figure 3 are conservative in the sense that they overestimate the shocks needed to destabilize countries. This is because we were focusing on the interaction between ratings and interest rates alone. All other variables that affect sovereign debt ratings were considered exogenous and thus kept constant during our simulations. In reality, the interest rate hikes that follow rating downgrades will increase budget deficits and debt ratios, and may depress economic activity in general. All this has an additional negative effect on a country's rating, and will thus reinforce the negative tendencies in the country's public finances. With this added transmission channel, even smaller shocks may already suffice to trigger sovereign debt crises. On the other hand, the simulations simplify in the sense that current interest rates are assumed to directly affect effective interest rates. A more detailed modelling of a country's debt structure might generate a smaller immediate effect from interest rates on ratings but it might generate an effect that lasts longer due to persistence in interest rates on the other hand. In that sense, the higher the average debt maturity of a country, the longer it takes until short term rating shocks affect the effective interest rate and thus feed back into the ratings.[31]

5.2 A note on the consistency of ratings

Have rating agencies made consistent judgements over time and between countries? As highlighted in the following paragraph, there is evidence in the literature that disputes this. It appears hard to find an algorithm that consistently arrives at the same conclusions as rating agencies do. However, the focus of this paper is not to contribute to this particular discussion. Rather, we draw on the findings in the literature about this phenomenon and analyse the consequences in a multiple equilibria setting. Then, we are able to show that ratings which potentially deviate from the true underlying probability of default might be justified later on due to a self-fulfilling prophecy mechanism.[32]

There is a large amount of evidence in the literature on inconsistency in sovereign debt ratings by the three big rating agencies. Gärtner et al. (2011) asked whether credit rating agencies applied the rating algorithm identified by equation 1 in Table 2 consistently

[31] This insight has also been emphasized in Cole & Kehoe (2000).

[32] In that sense, of course, there is no 'true' probability of default in this model of multiple equilibria. The nature of a self-fulfilling prophecy in this context is that an ex ante claimed probability of default in the future influences the validity of that same probability ex post. Then, of course, deviations of ratings from this non-existing true probability of default are hard to measure. In the words of Merton (1968) on self-fulfilling prophecies in general: "The self-fulfilling prophecy is, in the beginning, a false definition of the situation evoking a new behaviour which makes the original false conception come 'true'. The specious validity of the self-fulfilling prophecy perpetuates a reign of error. For the prophet will cite the actual course of events as proof that he was right from the very beginning."

over time and between countries, or whether some countries were singled out for special treatment, for better or worse. They find that the so-called PIGS countries, i.e. Portugal, Ireland, Greece and Spain, were indeed treated much more harshly in the aftermath of the financial crisis 2008-2009 than the remaining OECD countries and more harshly than they themselves where treated prior to Europe's debt crisis.

In line with these findings are the contributions by De Grauwe & Ji (2012*b*). Similarly, Ferri et al. (1999), Perrelli & Mulder (2001) and El-Shagi (2010) find inconsistencies in ratings during the East Asian Crisis in the late nineties of the past century.

Of course all these studies, including our own, may suffer from an omitted variable bias. Since rating agencies do not publish their rating methodologies in detail, one may argue that an important omitted variable led to a specific down- or upgrade. This can be brought forward, in particular, if one lists 148 groups of variables that potentially have an influence as Fitch, Inc. (2011) does. As long as rating agencies do not disclose their rating methodology in detail and explain their actions quantitatively instead of verbally, we are left with what economic theory suggests on this topic.

The fact that rating agencies are not impeccable in general has become evident during the recent financial crisis. Mortgage backed securities and other structured financial instruments with top ratings defaulted in large quantities (Crotty 2009). Even rating agencies themselves nowadays admit that they made errors during that period (BBC 2009).

5.3 Potential implications

To the extent that Figure 2 provides useful descriptions and simplifications of the structure, equilibria, dynamics and comparative statics underlying the eurozone's sovereign debt crisis, such crises may stem from two sources:

1. The estimated relationships are stochastic. Any deviation or change, as reflected in the error term of the estimation equations, may drive the country out of an initially stable neighbourhood and trigger a crisis. Section 5.1 looked at rating shocks and the damage they might do.

2. Any changes in the fundamentals that affect the positions of the i line or the r line and, thus, the equilibria, may make a country more vulnerable to a sovereign debt crisis. For instance, any change that increases the risk-free rate, moving the i line up, or changes that shift the rating line to the right, have two unfavourable effects. First, they render the good equilibrium less 'good', raising the associated interest rate and credit rating. Second, they move the insolvency threshold to the left, increasing the risk of being pushed into bankruptcy by unfavourable developments. If the relative effects are strong enough, the two curves may lose contact, making the good equilibrium and the threshold disappear, and rendering bankruptcy unavoidable.

Section 5.1 looked at rating shocks and suggested that they may have played a major role in the gestation and propulsion of Europe's sovereign debt crisis. A rating shock of 3.3 notches, to which the PIGS countries were subjected at the start of the crisis, according to Gärtner et al. (2011), would constitute a serious threat to all but the most highly rated countries. At the beginning and during the crisis, however, these risks were aggravated by budgetary and income shocks instigated by the housing and financial crisis. Coefficients from regression 1 in Table 2 permit a first quantitative assessment of the impact of these shocks on individual countries. A look at individual PIGS countries reveals the following. Deteriorating fundamentals shifted Portugal's rating curve to the right by 0.62 rating notches between 2009 and 2011. However, the country was downgraded by 8 notches during that time. For Ireland the line shift was 0.95 during those years, but the rating dropped by 7 notches. Greece's rating curve even improved by shifting 0.37 notches to the left, whereas the country had to deal with a hefty downgrade of 12 notches. Spain, finally, was downgraded by three notches from AA+ to AA-, while its rating line basically stayed put as it shifted 0.07 notches to the left.[33] In the context of the results reported in Figure 3, this suggests that budgetary and income shocks may have played a minor role only, and that exceptional changes in the risk assessment of the markets were a key factor in Europe's debt crisis.

6 Summary and conclusions

This paper analysed the European sovereign debt crisis that grew out of the global real estate and financial crisis of 2007-2009. Drawing on data for 25 OECD countries for the period between 1999 and 2011, we specifically asked whether there was evidence of multiple equilibria and self-fulfilling prophecy in the market for sovereign bonds. Special attention was given to non-linearities and dynamics in the interaction between government bond yields and sovereign bond ratings.

We find robust evidence of a non-linear relationship between ratings and interest rates that reflects the theory. This non-linearity is strong enough to generate multiple equilibria. This, in turn, may render rating errors or market panic stemming from other sources self-fulfilling in a strict sense. In the implied good and stable equilibrium, ratings are excellent and interest rates are low. A second equilibrium looms, which is unstable. It constitutes a threshold beyond which the country falls into an insolvency trap from which it may only escape by exogenous policy measures or outside intervention.

A more detailed look at the dynamics of rating downgrades revealed that, at least for countries with sovereign debt ratings outside the A range, even small, arbitrary rating downgrades may easily generate the very conditions that do eventually justify the rating.

[33]For this analysis we computed predicted values from regression 1 in Table 2 with actual country data. To focus on the impact of exogenous variables only, we froze the lagged average interest rate to its 2009 value.

Combined with earlier evidence that many of the rating downgrades of the eurozone's peripheral countries appeared conspicuous and could not be justified on the basis of rating algorithms that explain the ratings of other countries, or ratings in general before 2009, this result is discomforting. It urges governments to take a long overdue close look at financial markets in general, and at sovereign bond markets in particular, and at the motivations, dependencies and conflicts of interest of key players in these markets.

7 Appendix

7.1 Rating conversion

Table 5 shows the conversion of ratings into an equidistant numerical scale following other studies, such as Afonso et al. (2007).

Table 5: Rating conversion.

Rating	Numerical value
AAA	1
AA+	2
AA	3
AA-	4
A+	5
A	6
A-	7
BBB+	8
BBB	9
BBB-	10
BB+	11
BB	12
BB-	13
B+	14
B	15
B-	16
CCC+	17
CCC	18
CCC-	19
CC	20
C	20
RD	21
DDD	21
DD	21
D	21

7.2 Unit root tests

Table 6 shows the results for several unit root tests on all variables used in this study. ADF is the Augmented Dickey-Fuller unit root test applied to the pooled data set. The null hypothesis is that the series as a whole, has a unit root. IPS and LLC denote unit root tests proposed by Im et al. (2003) and Levin et al. (2002) specifically for panel data. The latter test can be applied for no intercept and an individual intercept, i.e. for fixed effects. IPS can only be applied to an individual intercept. The number of lags was automatically determined by the Schwarz Information Criterion (SIC). In the latter three tests, the null hypothesis is that each individual time series has a unit root. The values in the table are the respective statistics, i.e. Chi-square, Shin's W, t and Chi-square from left to right. The stars indicate the significance level of the respective test, i.e. 10%, 5% and 1%.

As the classic ADF test as well as most tests with no intercept reject the null, the use of a pooled OLS estimation should be possible. Also, using fixed effects should not be too worrisome considering only isolated acceptances of the null. Note, however, that there is some disagreement in the results of these tests, even within tests that evaluate the same null hypothesis as is pointed out in Verbeek (2008). All tests have their pros and cons. Furthermore, each of them relies on a number of underlying parameters, e.g. the selection of lags or bandwidth.[34] Nevertheless, several robustness checks are shown in the following sections, such as cross sectional and differenced time series estimates, in order to minimize the risk of invalid estimates.

Table 6: Unit root tests.

	ADF (stacked)	IPS (intercept)	LLC (none/intercept)
Foreign cur. rating (r)	−4.584***	0.904	−1.810*** / −2.694***
Local cur. rating (r_L)	−4.489***	−0.099	−0.0297 / −4.407***
GDP growth	−7.633***	−6.05***	−6.009*** / −3.707***
GDP per capita	−4.465***	−0.751	15.995*** / −3.515***
Budget surplus	−5.26***	−6.348***	−2.671*** / −2.262**
Primary surplus	−5.536***	−8.045***	−6.146*** / −4.797***
Debt ratio	−4.904***	−3.51***	1.877* / −6.656***
Inflation	−4.646***	−13.216***	−2.467** / −7.251***
Bond yield ($\bar{i}$)	−5.158***	1.549	−3.935*** / 2.114**
Credit spread ($i - i_D$)	−5.78***	−2.608***	−0.92 / 11.448***
Expected currency gains (ϵ_D)	−4.912***	−3.762***	−2.591*** / 0.453

7.3 Robustness checks on the rating equation

This section illustrates several robustness checks that have been performed in order to manifest the r-line regressions shown in Table 2.

7.3.1 Standard errors

Table 7 presents the same results as in Table 2 but displays normal standard errors. The Durbin Watson statistic indicates serial correlation in the error terms.[35] Table 2 in the text, therefore, shows standard errors corrected for heteroscedasticity and autocorrelation (hac) as documented in Zeileis (2004) which builds upon Newey & West (1987). Differences in the standard errors are rather small. Conventional p-value levels do not change at all or by one level in selected cases, i.e. *Primary surplus*, and *Inflation*.

7.3.2 Fixed effects

Table 8 presents analogous estimates of Table 2 but uses country fixed effects. A Breusch-Pagan Lagrange multiplier test suggests to make use of the panel structure, whereas a subsequently employed Hausman test indicates a preference of fixed over random effects.[36] The table shows heteroscedasticity and serial correlation consistent standard errors for within-group estimators as suggested by Arellano (1987).

[34] All unit root tests were performed using the *purtest* function of the *R*-package *plm* by Croissant & Millo (2008). IPS and LLC tests for foreign and local currency ratings were performed using the *Eviews* Software since the latter function produced no results due to short time series. Note that the data set consists of 11 years and 25 countries.

[35] A Breusch-Godfrey/Wooldridge test for serial correlation in panel models also rejects the null hypothesis of no serial correlation in the error term. See the *pbgtest* function of the *R*-package *plm* by Croissant & Millo (2008) for details on this test.

[36] We also checked for time as well as time and country fixed effects but both did not change the picture significantly.

Table 7: Regressions explaining sovereign debt ratings with normal standard errors.

	(1)	(2)	(3)
Constant	1.214	1.886*	1.723
	(0.966)	(1.059)	(1.056)
GDP growth	−0.049	−0.050	−0.044
	(0.065)	(0.064)	(0.065)
GDP per capita	−0.118***	−0.113***	−0.117***
	(0.016)	(0.016)	(0.016)
Budget surplus	−0.013	−0.021	−0.015
	(0.025)	(0.026)	(0.025)
Debt ratio	0.022***	0.020***	0.021***
	(0.003)	(0.004)	(0.004)
Primary surplus	−0.141***	−0.126***	−0.135***
	(0.037)	(0.038)	(0.038)
Inflation	0.178**	0.191**	0.187**
	(0.075)	(0.076)	(0.076)
$\bar{i}_{-1}$	0.693***	1.013***	0.541***
	(0.091)	(0.228)	(0.157)
$\log(\bar{i}_{-1})$		−1.469	
		(0.961)	
$\bar{i}^3_{-1}$			0.001
			(0.001)
R^2	0.608	0.584	0.610
adjusted R^2	0.598	0.574	0.599
Durbin-Watson	0.714	0.728	0.734
F	62.709	56.788	55.124
p	0.000	0.000	0.000
Observations	291	291	291

Notes: Pooled OLS regressions. The dependent variable is *foreign currency rating* r. $\bar{i}$ denotes government bond yields. Standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

Results, in general, do not differ much from the ones in the text. Growth, income per capita, budget surplus and debt ratio remain stable though significance levels change. The linear relationship with respect to the lagged average interest rate is affirmed, both, in terms of the size and the confidence level of the coefficient. We also conducted the Davies (1987) test on the fixed effects model and a constant coefficient with respect to the interest rate could again not be rejected. The most notable difference is that primary surplus switches the sign which is against intuition and to which we do not have a clear answer, unfortunately.

Table 8: Regressions explaining sovereign debt ratings with country fixed effects.

	(1)	(2)	(3)
GDP growth	−0.077	−0.079	−0.087
	(0.071)	(0.070)	(0.071)
GDP per capita	0.026	0.028	0.037
	(0.036)	(0.033)	(0.034)
Budget surplus	−0.074*	−0.074*	−0.074*
	(0.043)	(0.042)	(0.040)
Debt ratio	0.047***	0.047***	0.046***
	(0.013)	(0.013)	(0.012)
Primary surplus	0.221***	0.224***	0.226***
	(0.076)	(0.074)	(0.072)
Inflation	−0.074	−0.074	−0.075
	(0.069)	(0.067)	(0.064)
$\bar{i}_{-1}$	0.415**	0.317	0.620***
	(0.199)	(0.427)	(0.187)
$\log(\bar{i}_{-1})$		0.538	
		(1.558)	
$\bar{i}^3_{-1}$			−0.002
			(0.001)
Country FE	*Y*	*Y*	*Y*
(within) R^2	0.549	0.550	0.555
(within) adj. R^2	0.489	0.488	0.492
Durbin-Watson	1.351	1.350	1.336
F	45.128	39.436	40.258
p	0.000	0.000	0.000
Observations	291	291	291

Notes: Fixed effect regressions. The dependent variable is *foreign currency rating r*. $\bar{i}$ denotes government bond yields. Arellano (1987) heteroscedasticity and serial correlation robust standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

7.3.3 Generalized additive model

We also fitted a generalized additive model (GAM) to the data.[37] GAM models are generalized linear models in which the linear predictor is given by a sum of smoothing functions of the regressors instead of simple coefficients. Here we use penalized regression splines with four degrees of freedom. The estimated results are shown in Table 9. Figure 4 highlights once more a linear relation between the interest rate being paid on government debt and ratings. It also shows that some variables might also have a non-linear effect

[37] See Hastie & Tibshirani (1990)

on ratings, i.e. the debt ratio.[38] Due to the latter phenomenon we re-estimated the equations including a non-linear debt ratio as an additional explanatory variable. The results are shown in Table 10 and do not differ too much from previous ones. Non-linear debt is indeed significant, at least on the 10% level, in contrast to linear debt. This reflects theory in the sense that the net worth of a debtor has a nonlinear effect on the probability of default. The debt ratio, of course, is not a perfect proxy for the net worth of a country. Also it is not clear whether the observed values of this proxy are in areas in which nonlinearity of net worth already plays a role.[39] Since results do not improve significantly we stick to the simpler form in the text.

Table 9: GAM regresssions explaining sovereign debt ratings.

	edf	p-value
s(GDP growth)	2.969	0.000***
s(GDP per capita)	2.780	0.000***
s(Budget surplus)	1.000	0.909
s(Debt ratio)	2.978	0.000***
s(Primary surplus)	1.000	0.021**
s(Inflation)	1.380	0.030**
s($\bar{i}_{-1}$)	1.000	0.001***
adjusted R^2	0.784	
Durbin-Watson	0.831	
Observations	291	

Notes: Genaralized additive model. The dependent variable is *foreign currency rating r*. $\bar{i}$ denotes government bond yields. Standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. edf stands for effective degrees of freedom.

7.3.4 Cross sectional analysis

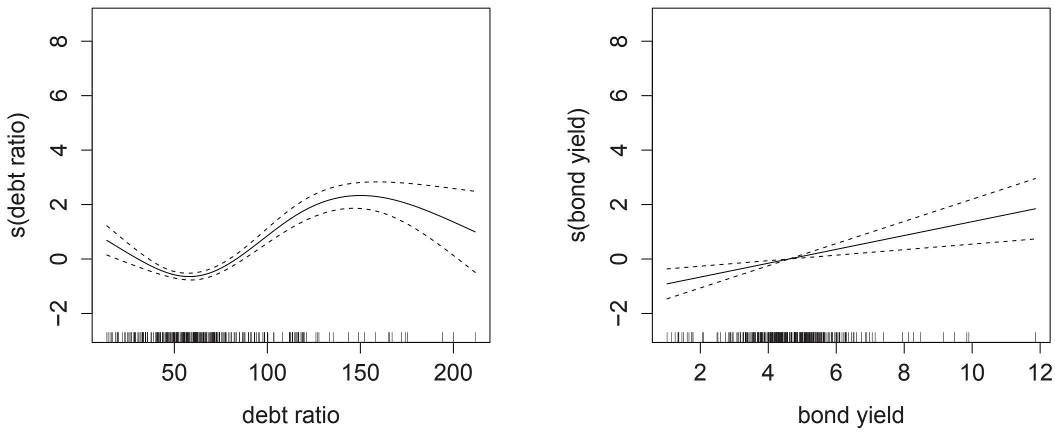
Table 11 re-estimates the regressions of Table 2 but uses only one year of observations, namely 2010. Of course, this reduces the number of observations to a very low level. Yet, it shows that in the absence of autocorrelation, the findings are rather robust. Significance levels of course drop due to the small sample. The model selection with respect to the linearity in interest rates can be affirmed. We also performed regressions with other years and small selections of years, such as 2005 and 2011 and the results did not change significantly.

7.4 Robustness checks on the interest rate equation

This section illustrates several robustness checks that have been performed in order to manifest the i-line regressions shown in Table 3 and 4.

[38] Results of GAM models have to be taken with care, however, since they are prone to over-fitting the data.

[39] We also tested for other nonlinear functional forms but they impaired the results.



(a) Spline estimation of the debt ratio. (b) Spline estimation of the (lagged) bond yield $\bar{i}_{-1}$.

Figure 4: Spline estimations of selected variables from the GAM regression shown in Table 9. Thin lines show 90% confidence bands.

Table 10: Regressions explaining sovereign debt ratings with nonlinear debt ratio.

	(1)	(2)	(3)
Constant	2.228	2.021	2.352
	(2.132)	(2.224)	(2.240)
GDP growth	−0.084	−0.086	−0.082
	(0.117)	(0.117)	(0.116)
GDP per capita	−0.112***	−0.114***	−0.112***
	(0.035)	(0.036)	(0.036)
Budget surplus	−0.032	−0.030	−0.033
	(0.057)	(0.058)	(0.058)
Debt ratio	−0.015	−0.017	−0.014
	(0.019)	(0.021)	(0.019)
Debt ratio^2	0.000*	0.000	0.000*
	(0.000)	(0.000)	(0.000)
Primary surplus	−0.115	−0.119	−0.114
	(0.074)	(0.079)	(0.076)
Inflation	0.215*	0.212*	0.216*
	(0.110)	(0.109)	(0.114)
$\bar{i}_{-1}$	0.724***	0.584	0.681**
	(0.207)	(0.462)	(0.329)
$\log(\bar{i}_{-1})$		0.659	
		(2.173)	
$\bar{i}^3_{-1}$			0.000
			(0.002)
R^2	0.629	0.629	0.629
adjusted R^2	0.618	0.617	0.617
Durbin-Watson	0.714	0.710	0.720
F	12.148	16.418	12.925
p	0.000	0.000	0.000
Observations	291	291	291

Notes: Pooled OLS regressions. The dependent variable is *foreign currency rating r*. $\bar{i}$ denotes government bond yields. Heteroscedasticity and autocorrelation robust standard errors in parentheses as documented in Zeileis (2004). *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

Table 11: Regressions explaining sovereign debt ratings in a cross sectional analysis

	(1)	(2)	(3)
Constant	−3.686	−3.432	−5.148
	(3.587)	(4.232)	(4.221)
GDP growth	0.238	0.236	0.203
	(0.312)	(0.322)	(0.321)
GDP per capita	−0.047	−0.046	−0.048
	(0.058)	(0.060)	(0.059)
Budget surplus	−0.100	−0.105	−0.076
	(0.075)	(0.085)	(0.084)
Debt ratio	0.032**	0.031*	0.035**
	(0.012)	(0.015)	(0.014)
Primary surplus	0.116	0.119	0.107
	(0.154)	(0.160)	(0.157)
Inflation	−0.136	−0.117	−0.201
	(0.367)	(0.408)	(0.385)
$\bar{i}_{-1}$	1.381***	1.470*	1.830**
	(0.392)	(0.829)	(0.766)
$\log(\bar{i}_{-1})$		−0.458	
		(3.727)	
$\bar{i}^3_{-1}$			−0.004
			(0.006)
R^2	0.767	0.767	0.774
adjusted R^2	0.671	0.651	0.661
F	8.006	6.601	6.846
p	0.000	0.001	0.001
Observations	25	25	25

Notes: Pooled OLS regressions. The dependent variable is *foreign currency rating* r. $\bar{i}$ denotes government bond yields. Standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

7.4.1 Generalized additive model

Analogous to the rating equation in section 7.3.3, we fitted a generalized additive model (GAM) to find an non-parametric estimate of the functional form in the interest rate equation. The estimated results are shown in Table 12. Figure 5 highlights once more a nonlinear relationship between the current rating and the current credit spread on government bonds.

Table 12: GAM regresssions explaining current credit spreads.

	edf	p-value
s(r)	2.956	0.000***
adjusted R^2	0.806	
Durbin-Watson	0.638	
Observations	291	

Notes: Genaralized additive model. The dependent variable is *credit spread* $i - i_D$. r denotes *foreign currency rating*. Standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. edf stands for effective degrees of freedom.

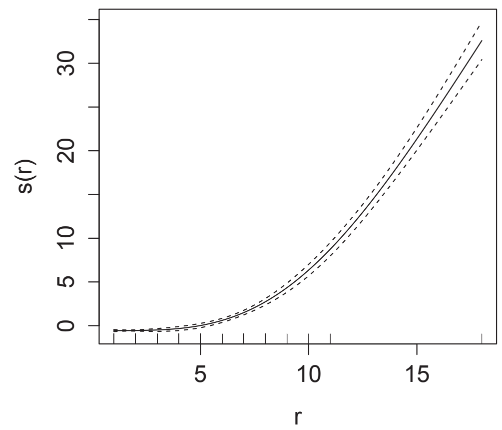


Figure 5: Spline estimation of the rating r from the GAM regression shown in Table 12. Thin lines show 90% confidence bands.

7.4.2 Coping with exchange rate risk

As stated in section 4 we abstain from adjusting the credit spread in the regressions shown in Table 3 and 4 for exchange rate risk since we use foreign currency ratings. An alternative approach might be to use local currency ratings and adjust the credit spread with respect to expected exchange rate changes. The results of this attempt are shown in Table 13. The regressions are analogous to the regressions 1, 4 and 6 in Table 3 and 4 but the explanatory variable switches from foreign to local currency ratings r_L and the dependent variable switches from credit spreads to credit spreads adjusted for expected currency gains, $i - i_D - \epsilon_D$. Coefficients and standard errors qualitatively do not change much. Regression 6 looses predictive power with respect to rating changes. Another notable difference is the decline in the R^2.

A potential downside of the estimations in Table 13 is the fact that long-term ratings have a different, potentially longer time horizon than the twelve month forward rates available. Also, it is not clear to which currency a foreign currency rating exactly refers to.[40] When comparing foreign and local currency ratings, it is obvious that they are mostly relevant for emerging economies. Both types of ratings are identical for all Euro countries, Norway, Sweden, the UK and the US in our time series selection.

Another, less questionable, robustness check is shown in Table 14 in which we limited the sample to Euro currency countries and thereby neutralize any influence of currency risk. Results do not change significantly with respect to the results shown in the text. One notable difference is that rating down- and upgrades now have symmetric effects.

Table 13: The effect of local currency ratings on sovereign bond yields adjusted for exchange rate risk

	(1)	(4)	(6)
Constant	−0.668	0.338	0.310
	(0.533)	(0.261)	(0.267)
r_L	0.825***	0.230*	0.243*
	(0.240)	(0.131)	(0.140)
r_L^3		0.005***	0.005***
		(0.001)	(0.001)
Δr_L^+			−0.021
			(0.409)
Δr_L^-			0.334
			(0.565)
R^2	0.292	0.469	0.473
adjusted R^2	0.290	0.466	0.466
Durbin-Watson	0.599	0.440	0.453
F	11.817	133.834	72.696
p	0.001	0.000	0.000
Observations	290	290	289

Notes: Pooled OLS regressions. The dependent variable is *credit spread* adjusted for the expected currency gains $i - i_D - \epsilon_D$. r_L denotes *local currency rating*. Heteroscedasticity and autocorrelation robust standard errors in parentheses as documented in Zeileis (2004). *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table. We dropped one observation in the last column since Greece fixed the Drachme to the Euro only on 19.06.2000.

7.4.3 Fixed effects

Table 15 shows fixed effects estimations of selected regressions from Table 3 and 4. A Breusch-Pagan Lagrange multiplier test suggests to use the panel structure and a

[40] Moody's Investors Service, Inc. (2006) states that foreign currency ratings reflect the "[...] opinion of the capacity and willingness of a government to mobilize foreign exchange to repay foreign currency-denominated bonds on a timely basis." whereas local currency ratings reflect the "[...] opinion of the capacity and willingness of a government to generate revenues in its own currency to repay its debt to bond holders on a timely basis."

Table 14: The effect of ratings on sovereign bond yields in Euro countries

	(1)	(4)	(6)
Constant	-1.553^{**}	0.021	0.157
	(0.683)	(0.168)	(0.147)
r	1.020^{***}	0.151	0.070
	(0.342)	(0.115)	(0.102)
r^3		0.005^{***}	0.004^{***}
		(0.000)	(0.000)
Δr^+			0.762^{***}
			(0.229)
Δr^-			0.703^{**}
			(0.322)
R^2	0.590	0.924	0.945
adjusted R^2	0.587	0.922	0.944
Durbin-Watson	1.077	1.189	1.896
F	8.908	902.076	7824.394
p	0.003	0.000	0.000
Observations	132	132	132

Notes: Pooled OLS regressions. The dependent variable is *credit spread* $i - i_D$. r denotes *foreign currency rating*. Heteroscedasticity and autocorrelation robust standard errors in parentheses as documented in Zeileis (2004). *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 11 Euro countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

Hausman test indicates to use fixed instead of random effects. Nevertheless, the results are again close to ones in the text.

7.4.4 Cross sectional analysis

Table 16 reproduces selective regressions from Table 3 and 4 in a cross sectional analysis. Here, only one year of observations, namely 2010, are used. Of course this reduces the number of observations considerably. Yet, it shows that in the absence of autocorrelation, the findings are rather robust. We also performed regressions with other years and small selections of years, such as 2005 and 2011, and the results did not change significantly.

7.4.5 First differences

According to the unit root tests in Table 6 the dependent variable in the regressions of Tables 3 and 4, i.e. the credit spread, might be non-stationary in a pooled OLS panel estimation. To deal with this issue, we took first differences of the respective variables and re-estimated the most relevant regressions. Table 17 shows the results which are still in line with the ones found in the text.

Table 15: The effect of ratings on sovereign bond yields with country fixed effects

	(1)	(4)	(6)
r	1.572***	0.395***	0.225
	(0.476)	(0.115)	(0.175)
r^3		0.005***	0.005***
		(0.000)	(0.000)
Δr^+			0.526
			(0.323)
Δr^-			0.107
			(0.096)
Country FE	*Y*	*Y*	*Y*
(within) R^2	0.661	0.897	0.904
(within) adj. R^2	0.602	0.813	0.814
Durbin-Watson	1.303	1.534	1.697
F	517.661	1144.387	614.856
p	0.000	0.000	0.000
Observations	291	291	290

Notes: OLS regressions. The dependent variable is *credit spread* $i - i_D$. r denotes *foreign currency rating*. Arellano (1987) heteroscedasticity and serial correlation robust standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

Table 16: The effect of ratings on sovereign bond yields in a cross sectional analysis

	(1)	(4)	(6)
Constant	−0.567	0.266	0.381
	(0.407)	(0.483)	(0.430)
r	0.726***	0.190	0.080
	(0.099)	(0.222)	(0.202)
r^3		0.006**	0.006**
		(0.002)	(0.002)
Δr^+			0.749**
			(0.283)
Δr^-			−
			(−)
R^2	0.700	0.771	0.828
adjusted R^2	0.687	0.750	0.804
F	53.623	37.092	33.813
p	0.000	0.000	0.000
Observations	25	25	25

Notes: Pooled OLS regressions. The dependent variable is *credit spread* $i - i_D$. r denotes *foreign currency rating*. Standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries in 2010. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

Table 17: The effect of ratings on sovereign bond yields with first differences

	(1)	(4)	(1)	(4)
Constant	0.146**	0.124***		
	(0.067)	(0.043)		
Δr	1.689***	0.346***	1.686***	0.344***
	(0.085)	(0.087)	(0.091)	(0.095)
Δr^3		0.004***		0.004***
		(0.000)		(0.000)
Country FE	*N*	*N*	*Y*	*Y*
R^2	0.601	0.839	0.587	0.825
adjusted R^2	0.596	0.830	0.530	0.741
Durbin-Watson	2.104	2.662	2.199	2.733
F	397.633	686.067	341.763	562.276
p	0.000	0.000	0.000	0.000
Observations	266	266	266	266

Notes: Pooled OLS and fixed effect regressions. The dependent variable is *credit spread* $\Delta(i - i_D)$. r denotes *foreign currency rating*. Standard errors in parentheses. *, **, *** denote significance at the 10%, 5%, 1% levels. Data for 25 OECD countries between 1999 and 2011. Ratings are transformed into an equidistant numerical scale from 1 (AAA) to 21 (D), see Appendix 7.1 for entire table.

Chapter 2

The role of corporate debt, risk and heterogeneity in the macroeconomy

I present a general equilibrium model with heterogeneous firms that exhibit uninsurable idiosyncratic productivity shocks and financing restrictions due to risk premia on external funding (loans). These arise due to costly defaults. Thus, firms face a trade-off in determining the amount of loans to be raised and the amount of dividends to distribute. Larger loans result in higher financing costs due to rising risk premia, but also in increased revenues due to the larger capital stock available for production. Growing internally through retained earnings means lower costs for external funds tomorrow, but also lower dividends to stockholders today.

The model is used to analyse the steady-state effects of shocks on aggregate productivity and on the volatility of idiosyncratic productivity shocks using comparative statics. Previous models in the literature have not been able to do so, either being partial equilibrium models or being based on exogenous default rates. The analysis also highlights the possibility of multiple equilibria when assessing risk premia and their consequences.

Keywords: incomplete markets, heterogeneous agents, firm behaviour

JEL: G3, D21

1 Introduction

The financial crisis that emerged in 2007 soon spread to the real economy. Many economists nowadays refer to this as a *credit crunch*.[1] The general lack of confidence prevailing in the money market quickly affected the credit market in the real economy because both markets are run by the same actors. If banks do not trust and lend to each other, how can they channel funds effectively to those who finally need it, namely, firms in the real sector? Thus, raising credit suddenly became difficult even for the healthiest companies

[1] See for example Whalen (2007), Campello et al. (2010) and Claessens et al. (2011).

in the real sector. Many economists did not anticipate this since debt on the supply side of the economy has tended to be seen as irrelevant. Typical general equilibrium models reduce the supply side of the economy to a representative firm that borrows its entire capital stock directly from households.

This paper presents a tractable general equilibrium model that captures important features missing in other models, such as firm heterogeneity, the capital structure of firms, borrowing constraints, idiosyncratic productivity shocks and endogenous risk premia. Given these features, it is possible to answer questions such as how does access to credit change the size distribution of firms, how does this distribution affect output or what is the optimal leverage ratio in an economy. The model is not calibrated to real world data, thus computational simulations can only present qualitative results. Nevertheless the model is a first step towards understanding in more detail how an economy produces its gross domestic product which might be an important ingredient for future policy analysis and welfare optimization.

In a nutshell, the model can be described as follows. Firms have decreasing returns to scale. Profits are either reinvested or distributed as dividends to stockholders. Up to an endogenous borrowing constraint, firms can also tap the credit market for loans. On the other side of the economy, households provide the labour force and save parts of their income. Savings can either be put into a bank account where the funds are redirected to firms as credit or they can be used to establish new firms. A general equilibrium is reached through wage adjustment: firms will be created as long as the return on stocks is larger than the return on the savings account. If so, the demand for labour increases as more firms are founded and wages rise. This reduces the profits of all firms and dividends fall. Consequently, the expected return from founding new firms shrinks and equilibrium is established.

Banks play a crucial role in handling risk in this setting. If a firm cannot pay back its loan due to an idiosyncratic productivity shock, it defaults. To compensate for this possible loss, banks charge a risk premium which depends on the capital structure of the firm, i.e. the ratio of loans to equity. Multiple equilibria can occur in this setting. Jumping between one and another has considerable effects: in a bad equilibrium with high interest rates, firms no longer have access to the credit market and are forced to grow internally. This affects the behaviour of firms, for example by increasing the amount of precautionary savings. The new general equilibrium results in a different size distribution of firms, lower wages, fewer firms and less output.

The model is solved numerically. It is not calibrated to real world data, but computational experiments present several qualitative results on comparative statics such as the effect of higher volatility in the production function or higher aggregate productivity. The paper also highlights that studying the size distribution of firms is indeed relevant and sometimes gives counter-intuitive results. If firms produce with decreasing returns to scale, aggregate output depends largely on the distribution of firms, as many small firms will produce more than a few large firms, for example.

The remaining part of this text is structured as follows. The next section gives an overview of the existing literature in this field. Thereafter, the model and agents are explained. Section 4 solves for the steady-state equilibrium, Section 5 shows the results of comparative statics and Section 6 concludes.

2 Related work

Analysing corporate debt in a heterogeneous agent, general equilibrium setting is a rather new field. Ultimately, this results from a lack of tools to solve such models for a long time. However, outside that framework, there is a vast body of literature that analyses corporate debt. Among the first authors to do so were Modigliani & Miller (1958) who showed that capital structure does not matter if markets are complete and if there are no distortionary taxes. In such a setting, firms can fully insure against any idiosyncratic risk through so-called Arrow-Debreu securities.[2] The simplest example of such a security is debt without borrowing constraints. Then, as idiosyncratic risk no longer plays a role, decisions concerning the capital structure of the firm become irrelevant. There is complete indifference among firms whether to retain earnings or distribute them as dividends and run into debt in order to maintain the amount of capital needed for production. All firms would then act in exactly the same manner, irrespective of their size. The use of the representative firm would be justified.

A large share of the literature focusing on corporate debt levels and corporate leverage relaxes the assumption of complete markets[3] by introducing financial frictions.[4]

One prominent example of financial frictions, is the costly state verification (CSV) problem introduced by Townsend (1979) which relies on asymmetric information: lenders cannot observe the performance of a borrower without cost. In fact, there is no need to do so, as long as a loan is paid back. If a borrower defaults, however, lenders have to analyse, at a certain cost, the true outcome of the investment project of the borrower in order to liquidate any remaining assets. This cost is usually charged to the borrower through a higher financing premium.

Bernanke & Gertler (1989) use the CSV approach in an overlapping generations model to show how productivity shocks to the economy are amplified and propagated. The shocks reduce net worth or equity respectively, leading to a higher share of external financing that increases agency costs, and thereby financing premiums, to an even greater extent. In total, this results in declining investment and even lower economic activity.

[2]See Arrow & Debreu (1954).

[3]There is probably an even larger body of literature focusing on consumer behaviour in incomplete markets. This field was introduced by Bewley (1977) and Huggett (1993) and then extended by Aiyagari (1994) and Krusell & Smith (1998). In Chatterjee et al. (2007) households can obtain unsecured credit and decide endogenously when to default.

[4]To the best of my knowledge, there is no formal definition of financial frictions. The survey by Brunnermeier et al. (2011) gives a comprehensive, although as yet incomplete, overview of the field.

In the literature, this phenomenon is commonly referred to as the *financial accelerator* effect. In Bernanke et al. (1999) this effect is shown in a more comprehensive New Keynesian model. However, both models rely on an exogenous probability of default, a simplification I eschew. Also, Bernanke et al. (1999) use a representative firm approach which does not allow them to comment on the distribution of firms with respect to their size and capital structure.[5]

The finance literature also covers bankruptcies due to insolvency and corresponding risk premia. However, this literature is typically restricted to partial equilibrium settings. The most outstanding articles in this field are, for example, Longstaff & Schwartz (1995) and Leland & Toft (1996).

This paper is closely related to the work of Cooley & Quadrini (2001). They show, in a partial equilibrium setting, how firms with decreasing returns to scale finance their capital needs either by raising debt, retaining earnings or issuing new shares. Moreover, they add financial frictions by introducing default costs and sunk costs when raising equity. As a result, they can replicate previous empirical results, for example those showing that larger firms are characterized by lower growth rates, lower default rates, higher dividends and less leverage. Cooley & Quadrini (2006) extend the previous model by embedding it in a general equilibrium setting and incorporating a monetary policy institution. This, however comes at the cost of omitting endogenous defaults.[6]

The results of financial frictions in the literature are often shown to be amplification or persistence effects as a consequence of aggregate shocks. These models become fairly complicated due to the nature of heterogeneity. Often these models are simplified when it comes to the formulation of defaults and risk premia.

This paper avoids this simplification. Therefore, the discussions are limited to steady-state analysis.

3 Model

The model describes a single good economy with three types of agents: a representative household, a representative bank and J heterogeneous firms.

3.1 Households

The representative household maximizes the discounted present value of future instantaneous utilities

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{1}$$

[5] Actually, Bernanke et al. (1999) do indeed introduce heterogeneity, but only in terms of two sectors that are different with respect to their production technology.

[6] Predecessors of Cooley & Quadrini in terms of modelling firm dynamics with endogenous capital structures under financial frictions are Clementi & Hopenhayn (2006) and Williamson (1987). All rely on financial frictions due to asymmetric information.

where $U(C_t)$ denotes the instantaneous utility of consumption at time t and β is the discount factor. The utility function U fulfils the Inada conditions. A household earns a real wage W on inelastic labour supply N. Savings can be invested in two types of assets: an investment fund and a savings account. The investment fund is an aggregate portfolio of stocks that pays dividends D_t. The savings account pays an interest rate of R. As a consequence, the household faces the following budget constraint:

$$C_t + B_{t+1} + I_t = (1 + R)B_t + D_t + WN \tag{2}$$

B_t denotes the stock of savings at the bank account. I_t denotes investment in the investment fund, hence, it is a flow variable. Increasing (or decreasing) the size of the investment fund results in the creation (or liquidation) of new firms and will thereby influence the flow of dividends D_t.

3.2 Banks

Banks collect deposits from households (savings account) and lend them to firms (loans). They pay households a risk-free rate R and charge each firm $j \in \{1, .., J_t\}$ an additional, individual risk premium $x_{j,t}$. As banks are perfectly competitive and homogeneous it holds that:

$$1 + R = (1 + R + x_{j,t})(1 - \lambda_{j,t}) + \mu\lambda_{j,t} \tag{3}$$

i.e. the risk premium $x_{j,t}$ compensates for a loss $1-\mu$ if the respective firm goes bankrupt and thereby defaults on its debt. A firm goes bankrupt with probability $\lambda_{j,t}$.

Since the decision on the amount and conditions of a loan requested by a firm is crucial for the results, the model makes an explicit distinction between households and banks, even though, from a theoretical point of view, there is no need for this distinction.

3.3 Firms

There are J firms which are heterogeneous in terms of size and capital structure. Each firm j is risk neutral and maximizes the expected present value of all future dividends d_t:[7]

$$\max \mathbb{E}\left[\sum_{t=0}^{\infty}\left(\frac{1}{1+R}\right)^t d_t\right] \tag{4}$$

A firm produces according to $y_t = y(z_t, k_t, n_t)$ where k_t denotes capital, $z_t \sim \mathcal{N}(1, \sigma)$, thus, $z - 1$ is an idiosyncratic productivity shock and n_t denotes the amount of labour employed. As in Cooley & Quadrini (2001, 2006), it is assumed that labour is a perfect complement to capital. For each unit of capital, γ units of labour have to be employed, or

[7] Throughout this text, upper case characters denote aggregates, whereas lower case characters denote per firm values. In what follows, the subscript j to denote an individual firm is omitted for the sake of notational simplicity.

$n_t = \gamma k_t$. This rather strong assumption holds without loss of generality, as long as there are no aggregate shocks in the economy. In a steady-state general equilibrium, wages are constant and therefore firms would pursue a constant capital-labour ratio even if the two factors were substitutable.

The production function can therefore be simplified by omitting labour and is assumed to have the functional form $y_t = z_t A k_t^{\alpha}$. The production parameter α is strictly smaller than one, thus, there are decreasing returns to scale at a firm level.

The amount of capital used for production can be financed through own funds, i.e. equity e_t, or external funds, i.e. loans l_t. Hence, the equality $k_t = e_t + l_t$ holds at all times. Equity can be accumulated by retaining earnings. Loans expand the amount of capital that can be used for production. If a firm takes out a loan, it has to pay the risk-free interest rate R plus an additional risk premium x_t which depends on the financial situation of the firm, namely, the size of the loan in comparison to its equity.

Each firm realizes a profit $\pi_t = y_t - (R + x_t)l_t - Wn_t - f$ where f is a fixed sunk cost. At the end of a period, before the dividend d_t has been paid out, the net worth of a firm, or equity before dividend, is denoted by $\tilde{e}_t = (1 - \delta)k_t - l_t + \pi_t$, i.e. capital stock after depreciation, less debt, plus earnings. Dividends are assumed to be $d_t \geq 0$ which is equivalent to saying that a firm cannot raise equity by issuing new shares but only by withholding profits.[8]

If the positive net worth condition $\tilde{e}_t > 0$ is violated, the firm goes bankrupt. In that case, the firm exits and the remaining value is zero. Outstanding loans are paid back only partially according to the recovery rate $\mu < 1$. In other words, this model features a financial friction in the form of costly default.[9] As mentioned above, this friction is typically justified using a costly state verification problem which was originally proposed by Townsend (1979). In the event of a default, the bank has to pay a monitoring cost to establish the true amount of remaining assets. Therefore, only a fraction of the entire amount $\tilde{e}_t + l_t$ can be paid back to the lender.[10]

The creation of a new firm is accomplished by providing an initial equity stock e^I and a fixed sunk cost f^I.

Figure 1 highlights the timing of events. A firm has two control variables: the amount of dividends, or the payout ratio, and the amount of debt. Both decisions affect the size of the capital stock that can be used for production. A crucial element in this decision

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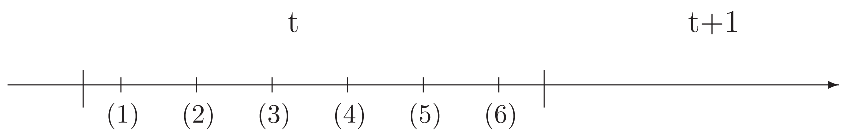
[8] Ross et al. (1993) and Smith (1977) empirically show that only a small share of equity is raised externally. Cooley & Quadrini (2001) introduce a cost when issuing new shares and thereby rule out this type of equity financing by adding another financial friction.

[9] If $\mu = 1$ there are no frictions and the Modigliani & Miller (1958) theorem applies.

[10] The default condition in this paper differs from the ones used in Cooley & Quadrini (2001, 2006). Firms in this paper have to declare default if they violate the positive net worth condition. This is a plausible assumption since in many advanced economies, e.g. Germany, this condition is state law (at least until the financial crisis 2008-2009). In the work of Cooley & Quadrini (2001) firms declare default if the continuation value becomes negative, i.e. if the discounted sum of all future expected profits is negative. In Cooley & Quadrini (2006) firms declare default at an exogenous exit probability i.e. independent of their economic situation.

is, of course, the risk premium. The higher the amount of debt, the more capital will be available for production. However, if the ratio of debt to equity becomes too large, the risk premium, and thus the expected profit, will shrink.



(1) Each firm with equity e_t decides on l_t and, thereby, on k_t.
(2) Each firm draws z_t, produces y_t and realizes a profit π_t.
(3) Firms with negative equity before dividend $\tilde{e}_t$ go bankrupt and exit.
(4) Continuing firms decide on d_t and thereby on e_{t+1}.
(5) Households collect earnings and decide on savings and type of investment.
(6) New firms are created.

Figure 1: Timing of events.

4 Optimal behaviour and equilibrium

The following subsections demonstrate the optimal behaviour of firms, banks and households. The last subsection defines the steady-state general equilibrium.

4.1 Optimal behaviour of the firm

The problem of the firm can be formulated recursively. The value of a firm with equity e_t is:

$$v(e_t) = \begin{cases} \max_{d_{z_t}, l_t} \mathbb{E}\left[\frac{1}{1+R}[d_t + v(e_{t+1})]\right] & \text{if } e_t > 0 \\ 0 & \text{if } e_t \leq 0 \end{cases} \tag{5}$$

subject to

$$e_{t+1} = \underbrace{z_t A k_t^\alpha - [1 + R + x(e_t, l_t)]\, l_t + (1 - \delta - \gamma W) k_t - f}_{=\tilde{e}_t} - d_t \tag{6}$$

$$k_t = e_t + l_t \tag{7}$$

$$l_t \geq 0 \tag{8}$$

$$0 \leq d_t \leq \tilde{e}_t \tag{9}$$

If equity falls below zero, the positive net worth covenant comes into play and $v = 0$. The risk premium x depends on the leverage of the firm, calculated by the amount of equity e_t and the size of outstanding loans l_t. The dividend d_t is chosen ex post the shock, which is the reason for the notation of d_{z_t} in the maximization problem. The budget constraint (9) denotes that a firm can distribute at most the available equity $\tilde{e}_t$ at the end of the period if it is positive. Both the dividend and the value of the firm in the next period have to be discounted by the interest rate, since dividends can only be used in the next period.

The shock z_t is not a state variable of the value function since shocks are independent and identically distributed. However, the value function clearly depends on the state of the economy, summarized by the wage rate W and the risk free interest rate R. Note that since there are no aggregate shocks in this economy, wage and interest rates are constant in equilibrium and no laws of motion for these variables are needed, as for example in Krusell & Smith (1998).

It is not possible to solve the problem of the firm analytically. As will be shown in Section 4.2, there is no closed form solution to the risk premium function with respect to e_t and l_t which would be needed to compute the necessary partial derivatives. Also, there is an (endogenous) borrowing constraint. Above a certain capital ratio, banks charge an infinite risk premium as they anticipate a large probability of default. It is well known from the literature on incomplete markets that models with idiosyncratic risk and borrowing constraints can in general not be solved analytically.[11]

For the numerical solution, it is helpful to convert the problem of the firm (5) into a sequential problem:

$$v(e_t) = \max_{l_t} \mathbb{E}\left[\frac{1}{1+R} w(\tilde{e}_t)\right] \tag{10}$$

$$w(\tilde{e}_t) = \begin{cases} \max_{d_t} d_t + v(e_{t+1}) & \text{if } \tilde{e}_t > 0 \\ 0 & \text{if } \tilde{e}_t \leq 0 \end{cases} \tag{11}$$

subject to

$$\tilde{e}_t = z_t A k_t^{\alpha} - [1 + R + x(e_t, l_t)]\, l_t + (1 - \delta - \gamma W) k_t - f \tag{12}$$

$$k_t = e_t + l_t \tag{13}$$

$$e_{t+1} = \tilde{e}_t - d_t \tag{14}$$

$$0 \leq d_t \leq \tilde{e}_t \tag{15}$$

At the beginning of the period, before a shock is drawn, the firm decides upon the size of the loan and thereby the amount of capital used for production. After the shock has been drawn and output has been produced, the firm either has to file for bankruptcy

[11] See, for example, Zeldes (1989) and Huggett (1993)

or, it continues. In the latter case, it has to decide how much to pay out in dividends and, therefore, how much equity to use in the subsequent period. This problem is solved numerically using value function iteration.

4.2 The risk premium

Finding the risk premium can be accomplished by solving a simple fixed point problem.[12] At the beginning of each period, a firm might ask a bank for a loan for this period. The bank provides this loan at a given risk premium depending on the size of the loan and the available equity. To simplify the analysis, the recovery rate is assumed to be $\mu = 0$.[13]

Let $\bar{z}$ denote the size of the idiosyncratic shock z that, given a specific capital structure e and l, leads to zero equity before dividends $\tilde{e} = 0$. Hence, $\bar{z}$ is the shock below which a firm goes bankrupt. It holds that

$$\tilde{e} = 0 = \bar{z}Ak^{\alpha} - (1 + R + x)l + (1 - \delta - \gamma W)k - f \tag{16}$$

given $k = e + l$.[14] Since the probability of default is $\lambda = \mathcal{P}(z \leq \bar{z})$ it is now possible to determine the cut-off shock as

$$\bar{z} = \Phi_z^{-1}(\lambda) \tag{17}$$

where Φ_z^{-1} denotes the inverse of the cumulative distribution function of z. From the no-profit condition of banks (3) the following equation for the default probability λ can be derived:

$$\lambda = \frac{x}{1 + R + x} \tag{18}$$

Using (17) and (18) in (16), it is possible to define a numerically solvable function

$$x = x(e, l) \tag{19}$$

that depends on W and R and that solves the equation

$$\Phi_z^{-1}\left(\frac{x}{1 + R + x}\right) Ak^{\alpha} - (1 + R + x)l + (1 - \delta - \gamma W)k - f = 0 \tag{20}$$

with $k = e + l$.

Equation (20), however, does not always have one unique solution. In fact, it may have zero, one or two solutions. If there is no solution, i.e. if the left-hand side (lhs), interpreted as a function of x, has no roots, then there are two cases that have to be distinguished:

1. lhs (20) $> 0 \; \forall x \Rightarrow \lambda = 0, \; x = 0$

[12] Calvo (1988) solves a similar problem when pricing public debt.

[13] This can, of course, be adjusted. However, it comes at the cost of more complicated equations.

[14] Time indices are omitted in this subsection, since the analysis of the risk premium is an intratemporal problem.

2. lhs (20) $< 0 \; \forall x \Rightarrow \lambda = 1,\; x = \infty$

In certain cases multiple equilibria occur, as shown in Figure 2 which plots the left-hand side of equation (20) with respect to different values of x or λ, respectively. For the benchmark case, the left most equilibrium is assumed, namely, that banks chose the lowest risk premium.

This equilibrium also allows banks to choose $x = 0$, if the left-hand side of equation (20) is positive for this value. That is, if equity remains positive for a loan with no risk premium even after the lowest possible shock, banks will ex ante assume a corresponding default probability of zero. Section 5.2 will relax on these assumptions and will also discuss the possibility of the "bad" equilibrium on the right of Figure 2 under rational expectations.

Also, the combination of $\lambda = 1$ and $x = \infty$, which is always a solution as long as $l > 0$, is only chosen if there is no other lower equilibrium.

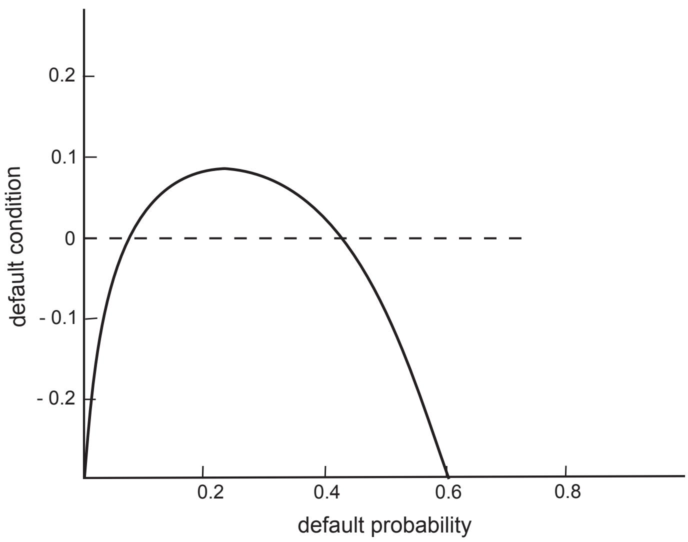


Figure 2: Multiple equilibria when determining the default probability and the corresponding risk premium.

Figure 3 shows risk premia for different combinations of e and l given a certain level of R and W. The picture clearly shows the positive correlation of leverage and risk premium. Increasing the amount of debt causes higher risk premia, ceteris paribus. Also, the figure clearly highlights that, the more dividends are paid out, the lower next period's equity, the higher the leverage and thus the higher the risk premium for a given amount of debt.

Given the numerical function to compute the risk premium, it is now possible to solve the problem of the firm (10)-(15) by value function iteration. Figure 4 shows the resulting policy functions for some given state of the aggregate economy, namely, for a given wage rate W and a risk free interest rate R. It also highlights the trade-off between taking up more debt and being faced with a higher risk premium. The dashed lines define the

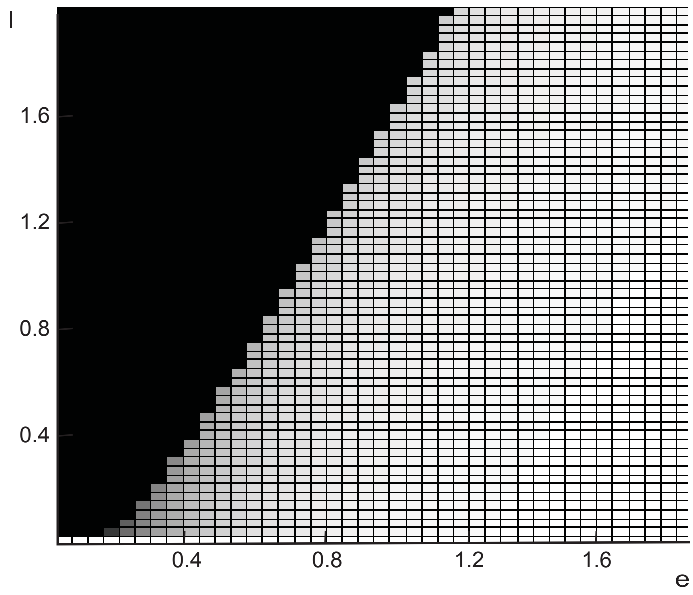


Figure 3: Risk premia with respect to equity and debt. The darker the region, the higher is the risk premium. Black areas correspond to an infinite risk premium or a default probability of one, respectively.

threshold at which the risk premium begins to rise above zero (lower dashed line) and at which the risk premium is one, hence at which no loans are granted (upper dashed line).

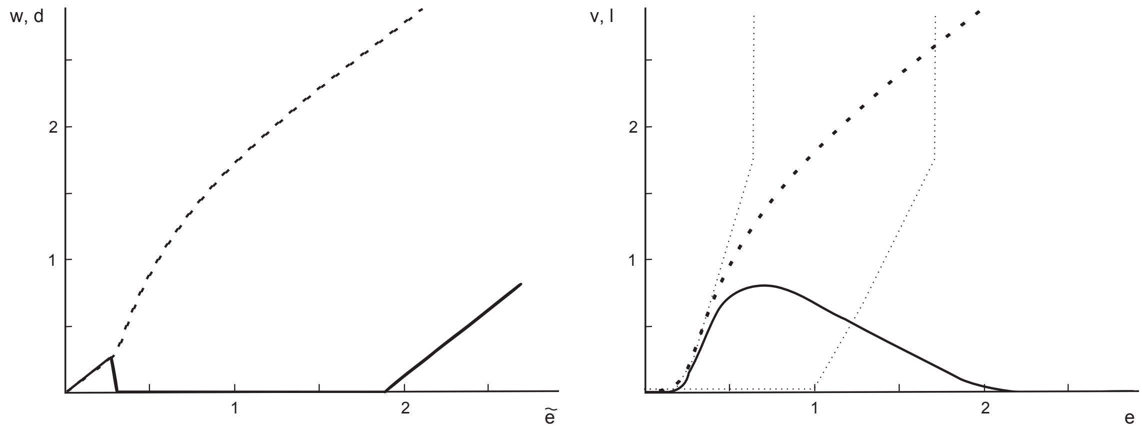
Several features should be noted: Below a certain equity threshold, the firm realizes that it is hopeless to continue. It does not have access to any foreign funds and the probability of default, due to the low production level, is so high, that it decides to distribute all equity as dividends and run into default in the next period.[15]. Above that threshold the firm suddenly has hope. It retains all equity and earnings and requests all external funds it can get in order to reach the optimal production size as fast as possible. However, external funds are bounded by the endogenous borrowing constraint. After growing steadily, the risk premium falls, and the firm eventually reaches the optimal production level of capital.

Once the firm has achieved the optimal capital stock, it starts distributing dividends. Due to the decreasing returns to scale production function, it would not make sense for the firm to employ any more capital than optimally needed. It is more profitable for the owner of the firm to put the dividends into a savings account or invest in new (small) firms instead. Therefore, if a positive shock pushes a firm above its desired capital stock, it will immediately distribute the excess amount of capital as dividends. If it falls below due to a negative shock, it will again stop paying out dividends and build up its own equity to move back to the optimal capital stock.

Figure 5 shows the convergence of a firm to a mean shock equilibrium. That is, the

[15]Note, that there is no damage to any party, since there are no loans outstanding and all equity has already been distributed

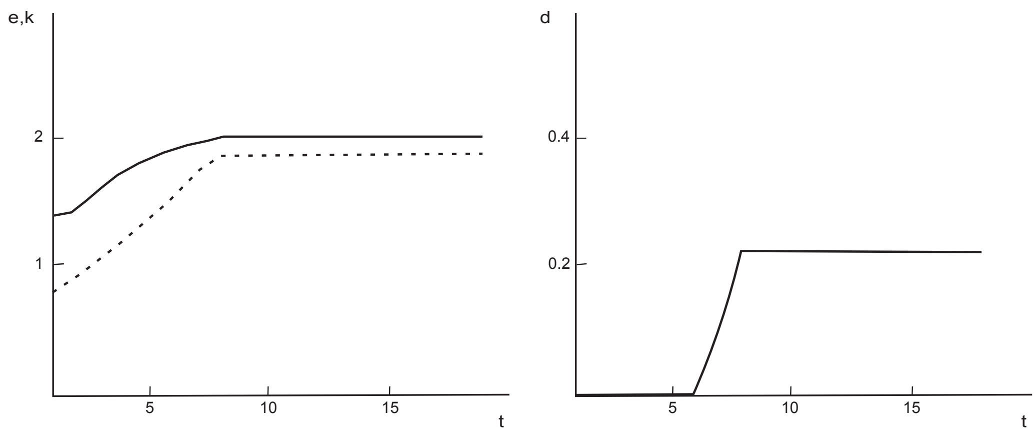
[17]Dividend and loan lines are smoothed.



(a) Optimal dividend policy (b) Optimal loan policy

Figure 4: Optimal policy functions. The left figure shows the value function w (dashed line), as well as the optimal amount of dividends (solid line) with respect to the current size of equity before dividends $\tilde{e}$. The right figure shows the value function v (dashed line) and the optimal loan (solid line) with respect to the current size of equity after dividends e. The dotted lines define the threshold where the risk premium begins to rise above zero (lower dotted line) and where the risk premium is one, hence, where no loans are possible (upper dotted line).[17]

behaviour of the firm is simulated for several periods where in each period the realization of the idiosyncratic shock is its mean. Starting at an arbitrary level of equity below the optimum, the firm takes up high loans in order to get as close as possible to the optimal capital stock. However, it cannot immediately grow to its desired capital stock since high interest rates impede arbitrary high loans. Dividends are not paid out, until the optimal size e^* has been reached. This reflects the results found in Cooley & Quadrini (2001).



(a) Capital (solid line) and equity (dashed line). The difference between both lines is debt. (b) Dividends

Figure 5: Mean shock firm equilibrium.

4.3 Optimal behaviour of the household

The household has to make two decisions: How much to save and where to put the savings. That is, should savings be put in a bank account or should they be used to establish new firms?

The first decision is a standard intertemporal optimization problem and can be solved analytically, for example by applying the calculus of variations as shown in Gali (2008). This leads to the well-known Euler equation:

$$U'(C_t) = \beta[(1+\hat{R})U'(C_{t+1})] \tag{21}$$

Here, $\hat{R}$ denotes the rate of return a household can achieve on its savings. This decision, is independent of the type of saving i.e. deposits or new firms or both.

An expectations operator is not needed here, since there are no aggregate shocks and each household holds a share of the portfolio of all firms. This makes the expected rate of return the true return by the law of large numbers.

In order to decide which type of investment the household would choose, it is necessary to calculate the expected rate of return from investing in a new firm R^I. This is determined by the expected value of the firm, which depends on the initial equity invested and on the sunk cost f^I.

$$R^I(e) = \frac{v_{W,R}(e)}{f^I + e} - 1 \tag{22}$$

The value function v depends on the state of the economy, namely the current wage rate W and the risk free interest rate R. Obviously, households maximize this rate of return, implying that the optimal initial equity denoted by e^I is:[18]

$$e^I = \arg\max_e \frac{v_{W,R}(e)}{f^I + e} - 1 \tag{23}$$

The calculation of the optimal steady-state entry size e^I according to (23) is highlighted in Figure 6 which plots the rate of return for each initial level of equity in a certain aggregate state given by W and R.

4.4 Steady-state equilibrium

The steady-state equilibrium is given by:

- A constant risk free interest rate that is ultimately determined by the household's

[18]Cooley & Quadrini (2001, 2006) use an additional exogenous success rate that determines the probability of successful firm creation. This reduces the initial firm size. Setting the success rate to one, as is done here, results in $e^* = e^I$. Firms will be established with their optimal size.

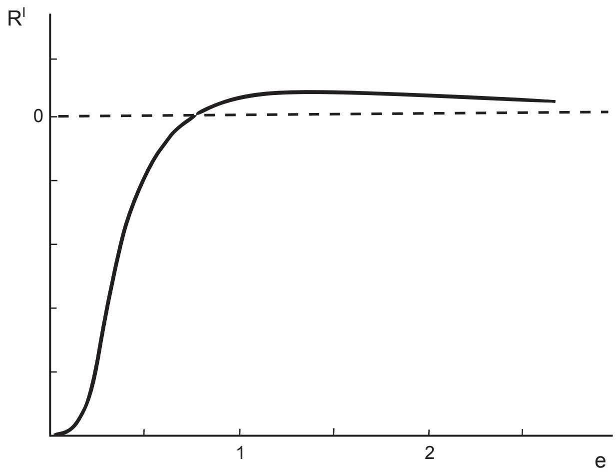


Figure 6: Rate of return from investing in a new firm with respect to the initial firm size.

discount rate and the Euler equation (21):

$$\hat{R}^* = \frac{1-\beta}{\beta} \tag{24}$$

- Constant and equal rates of return from deposits and stocks. If one of these were higher, households would increase investment in the more prosperous alternative and thereby raise the return of the other. Hence, $R^I = R$.

- A wage rate W^* that clears the supply and demand for labour such that $R^I = \hat{R}^*$. If at any time R^I was greater (lower) than $\hat{R}^*$, this would lead to a rise (fall) in the creation of new firms. Since labour is a limited factor in this model, the wage rate would rise (fall) due to increasing (decreasing) demand. This decreases (increases) the value function v and thereby decreases (increases) R^I.

- Optimal policy functions $d^*(\tilde{e})$ and $l^*(e)$ and corresponding value functions v and w that solve the problem of the firm given by (10) to (15).

- An invariant distribution of firms with respect to equity, debt and capital.

- A constant aggregate capital stock equal to $K^* = \gamma N$

- A constant inflow and outflow of firms due to the establishment of new firms and the bankruptcy of existing firms. In each period, it holds that the amount of capital used to create new firms K_t^{new}, and the change in the capital stock of surviving firms, K_t^{growth}, is equal to the amount of capital lost by firms that go bankrupt, K_t^{default}. Hence, whereas in general it holds that

$$K_{t+1} = K_t + K_t^{\text{growth}} + K_t^{\text{new}} - K_t^{\text{default}} \tag{25}$$

in a steady-state equilibrium

$$K^{\text{growth}^*} + K^{\text{new}^*} = K^{\text{default}^*} \tag{26}$$

4.5 Calibration

The model is solved numerically by the use of value function iteration and simulation. Details can be found in the Appendix 7.1.[19] For the numerical results, shown below, I use the exogenous parameters reported in Table 1. The parameters α, β and δ are set to typical values used in the literature of calibrated general equilibrium models. A and N can be varied without changing the results since they simply scale the size of the economy. The remaining parameters are in magnitude consistent with the values used in Cooley & Quadrini (2006). The goal of this paper is to show the qualitative effects of a heterogeneous supply side and its dependence on foreign and own funds on the economy, rather than matching real data.

Parameter	Value	Description
β	0.93	discount rate
α	0.4	production factor
δ	0.1	depreciation rate
f	0.7	fixed sunk production cost
f^I	0.7	fixed sunk investment cost
A	1.0	aggregate technology
γ	1.0	capital-labour ratio
σ	0.3	standard deviation of shock
N	5000	inelastic labour supply

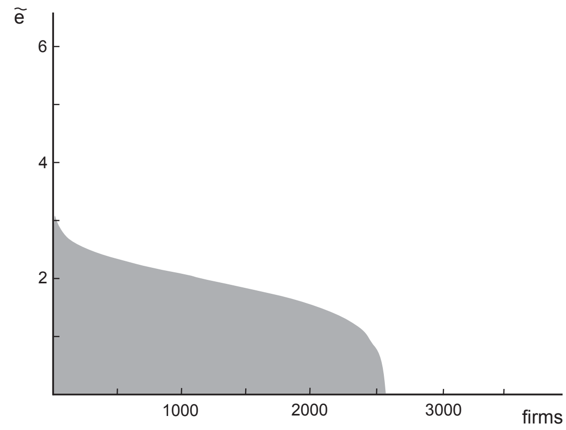
Table 1: Numerical parameters

Figure 7 gives an idea about the steady-state distribution of equity before and after the payment of dividends. Of course, after dividends have been paid out, all firms share a common maximum capital stock, i.e. the optimum. Any equity in excess of the optimum will be distributed as dividends. Figure 7(b) also shows the amount of loans outstanding for each level of equity. As expected, small firms take on higher levels of debt in order to reach the optimal capital stock faster.

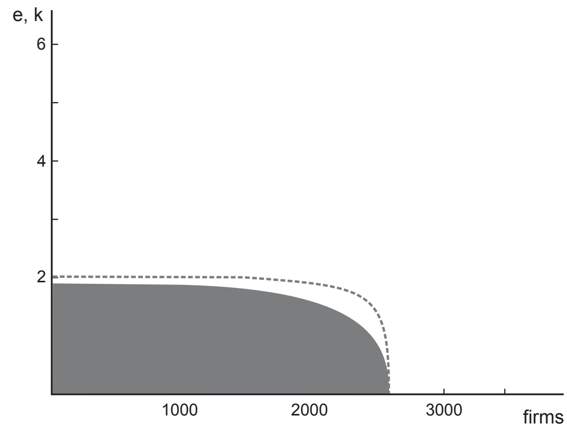
In the remaining part of this section, some insights with respect to the settings of the model shall be given.

The aggregate capital stock is fixed by construction since labour is supplied inelastically and capital and labour are perfect complements. Some sort of limited production factor, here labour, is needed for convergence. If there was no limited factor, the model would result in an AK Model that either grows forever or collapses. This is due to the

[19] Note that perturbation methods i.e. a log linearization around a steady-state as proposed in Blanchard & Kahn (1980) is not feasible in heterogeneous agent models as deviations from the steady-state are naturally quite large.



(a) Equity before dividend payment.



(b) Equity after dividend payment.

Figure 7: Equity of firms in the steady-state equilibrium. The dashed line in the right figure shows the subsequents period amount of capital employed. Thus, the difference between the dashed line and the boundary of the grey area denotes the amount of loans.

fact that decreasing returns at the firm level is not a sufficient condition for decreasing returns in the aggregate. Households could continuously create firms of size e^I without reducing R^I if the wage rate did not change.

Decreasing returns to scale are needed in order for firms to gain positive profits. Alternatively, one could use monopolistic competition, but the calculations are more cumbersome.[20] Profits are needed to give firms a choice concerning the dividend payout ratio, i.e. how much of the profits to reinvest in equity and how much to pay out. Fixed costs in the production function avoid that the optimal firm size approaches zero in the limit as returns would go to infinity.

Labour is the perfect complement to capital for the sake of simplification of the analysis. This reduces the problem of the firm by one choice variable. Relaxing this constraint would not change the results. The capital labour share would be constant but dependent on the distribution of firms if, for example, a Cobb-Douglas production function was used.

5 Comparative statics

This section analyses different steady-state scenarios. The first scenario looks at the effect of increased volatility of the idiosyncratic productivity shock. The second analyses the effect of banks choosing the "bad" equilibrium with high interest rates and high default probabilities. In the last part, the characteristics of a steady-state with an increased aggregate productivity are shown.

[20]Most models in this field use decreasing returns to scale such as those of Cooley & Quadrini (1999), Cooley & Quadrini (2006), Kiyotaki & Moore (1997), Covas & Den Haan (2010). Monopolistic competition is typically used in models with sticky prices as it is the case in New Keynesian models. An exception is the New New Trade Theory spearheaded by Melitz (2003)

All results are summarized in Table 2. The columns of the table report different statistics of the steady-state equilibrium: aggregate output, average leverage ratio measured as $\frac{l+e}{e}$, the minimum firm size (i.e. the amount of equity that influences whether the firm decides to continue or give up and distribute all remaining equity as dividends), the number of firms and the aggregate wage rate. The first row shows the baseline equilibrium statistics normalized to 100%. The rows below display the results of the different experiments of the following three subsections with respect to the baseline scenario.

Setting	Output	Leverage	Min. firm	# Firms	Wage
Baseline $\sigma = 0.30, A = 1.00$	100%	100%	100%	100%	100%
Increased volatility $\sigma = 0.60, A = 1.00$	95.8%	93.0%	89.6%	87.8%	70.5%
High interest rate equilibrium $\sigma = 0.30, A = 1.00$	98.9%	87.7%	152.3%	99.7%	96.8%
Increased productivity $\sigma = 0.3, A = 1.1$	126.3%	96.5%	88.6%	124.8%	176.8%

Table 2: Results of the comparative static analysis.

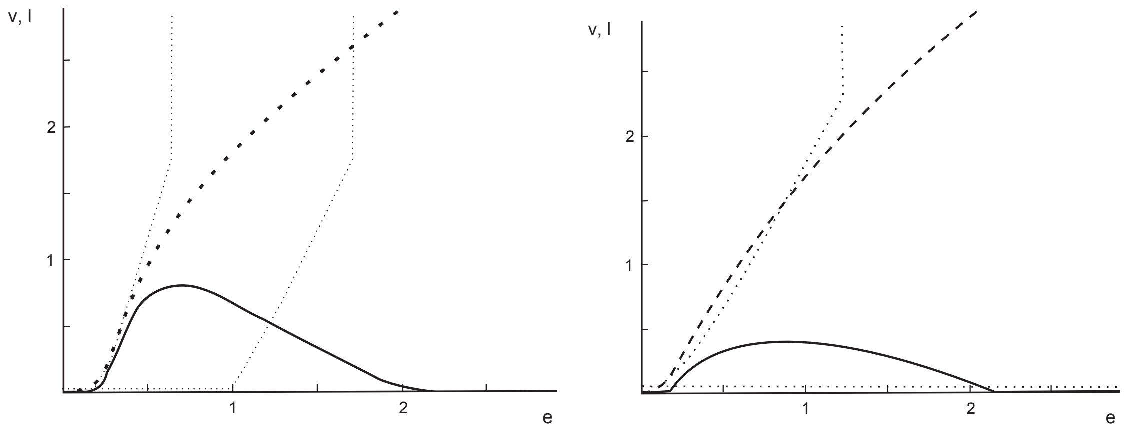
5.1 Idiosyncratic volatility shock

This section highlights the effect of a shock on volatility of idiosyncratic productivity. Therefore, the standard deviation from the baseline model is increased from $\sigma = 0.3$, to $\sigma = 0.6$. The policy function with respect to loans has quite different characteristics than before. As shown in Figure 8, firms take lower loans when they are small, due to higher risk premia. An analysis of the mean shock equilibrium also reveals that they do not maintain a positive loan level in their optimal state. This is due to the fact that the zero risk premium bound, i.e. the lower dashed line in Figure 8(b), shows that firms are always charged a positive risk premium.

In the aggregate equilibrium, this leads to a lower level of debt in the economy. Inequality in the size distribution of firms has increased, as shown in Figure 9. Of course, this is mostly driven by the higher variance of the shock itself. The figure also highlights the decline in the number of firms.

The aggregate results can be observed in the second row of Table 2. An increased idiosyncratic volatility leads, as expected, to a lower leverage ratio compared to the baseline setting, as it is harder for firms to access loans due to higher risk premia. There are fewer firms, indicating a higher average firm size which corresponds to the lower wage rate. Interestingly, the minimum firm size decreases, despite the larger volatility and thereby increased financing restrictions for small firms. Hence, the general equilibrium effect of lower wages overcompensates for the increased risk due to higher volatility.

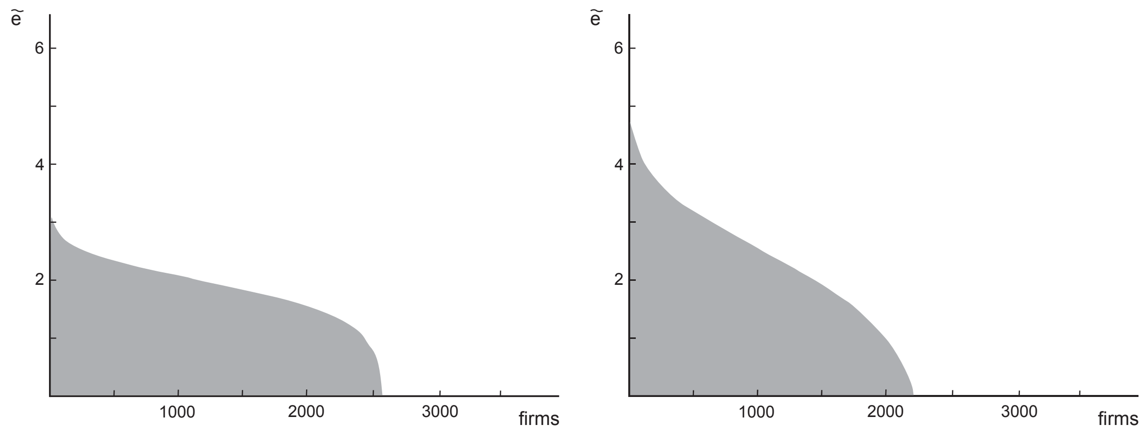
Note that a comparatively lower wage rate with respect to output, as is the case here, means that capital owners among the households profit from the higher rate of volatility.



(a) Optimal loan policy with $\sigma = 0.3$ (b) Optimal loan policy with $\sigma = 0.6$

Figure 8: Optimal loan policy (solid line) at different volatilities of the idiosyncratic shock. For comparison, the left side redraws the policy function of the baseline model shown in Figure 4(b). The dashed lines show the value function v whereas the dotted lines define the threshold where the risk premium begins to rise above zero (lower dotted line) and where the risk premium is one (upper dotted line).

Apparently, this model does not make an explicit distinction between capital owners and workers as there is just one representative household.



(a) Equity before dividend payment with $\sigma_b = 0.3$ (b) Equity before dividend payment with $\sigma_h = 0.6$

Figure 9: Equity of firms before dividend payments. For comparison, the left side redraws the distribution of the baseline model shown in Figure 7(a).

5.2 High interest rate equilibrium

Section 4.2 emphasized the possibility of multiple equilibria when banks assess the individual risk premium. As stated before, banks have chosen so far the "good" equilibrium in Figure 2, with low interest rates and small risk premia. A question worth analysing is whether banks might also choose the "bad" equilibrium and at what consequences? Answers with respect to the first question can be found in Calvo (1988) and Detragiache

(1996). The former claims that the "bad" equilibrium is not sustainable since any single creditor could gain excess returns by providing loans at a lower interest rate. More precisely, if the current interest rate is at 45% in Figure 2, a creditor asking for only a slightly lower interest rate, which then corresponds to a lower probability of default, would push all other creditors out of the market and gain excess returns. Consequently, in a competitive credit sector, the "bad" equilibrium is not possible. However, as Detragiache (1996) argues, this result relies on a crucial assumption: the credit market has to be populated by large investors and comparably small debtors. In the high interest rate scenario given above, deviation is profitable if and only if the entire credit amount can be provided by a single or small group of cooperating creditors. Otherwise, if the credit market is populated by many small investors, deviation is not profitable. Since the major amount of debt would still be provided by others, the effective interest rate and, thus, the default probability of the debtor remain high. As a consequence, the small deviating investor would experience losses in expectation. Boot et al. (2006) also arrive at this conclusion in a similar setting: they argue that multiple equilibria can occur and that rating agencies can serve as a coordinator among them. Using a rational-herd argument,[21] they show that if enough agents base their investment decisions on ratings, others rationally follow. This result holds for both equilibria, good and bad.

Figure 10 shows what happens if banks choose the high interest rate equilibrium. An economic justification for such a choice could be a certain anxiety in the economy, as has been observed during the credit crunch phase in the aftermath of the 2007 financial crisis and described by Brunnermeier (2009).

In Figure 10(a) it can be observed that the average risk premium increases to a very high level, as most of the area is shaded much darker than in the baseline scenario plotted in Figure 3. The reason for the bright area along the diagonal is as follows: with rising equity the maximum of the default condition shown in Figure 2 shifts from below the zero line, slowly upwards. At one point, this results in the emergence of only one single root, i.e. the maximum itself. In that case, the resulting risk premium is the same as before. However, with rising equity the maximum moves further upwards, causing the equilibrium interest rate on the right to increase and the one on the left to decrease.

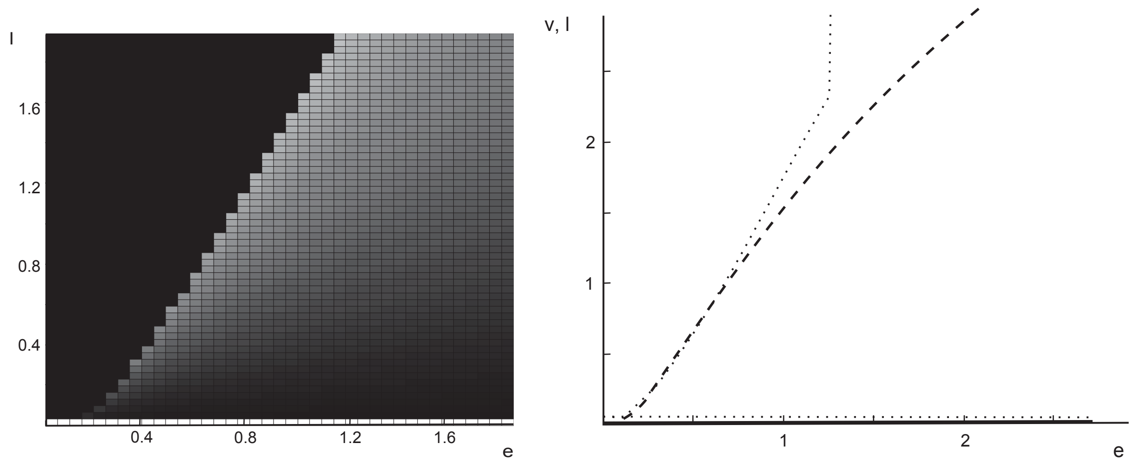
As a result of the higher risk-premium, a firm never takes on any debt, as can be observed in the policy function in Figure 10(b). The consequence for the general equilibrium is that small firms, which can emerge either due to a bad shock or because they were established with low initial equity, need much more time to reach the optimal capital stock.

Moreover, as can be observed in the third row of Table 2, this leads to a significantly higher minimum firm size, slightly lower output and a lower wage rate.[22] The number of

[21] See Chamley (2003) for an introduction to rational herds.

[22] Note that a larger minimum firm size is quite interesting if we think outside the boundaries of this model, in which all households perfectly coordinate their savings and create new firms if necessary. If that was not the case, further frictions could arise, namely, that each group of investors cannot raise

firms stays roughly the same, i.e. the average firm size should be similar to the baseline model. This is a surprising result since the more restrictive borrowing constraint should raise precautionary savings and thereby the optimal equity stock.[23] This effect, plus the larger minimum firm size should shift the average firm size upwards. The reason, for not observing this, is again due to the general equilibrium effect resulting from the lower wage rate.



(a) Risk premia are very high (dark) in almost every region.

(b) A firm never takes on any debt.

Figure 10: The effect of the high interest rate equilibrium. In the right figure, the optimal loan, denoted by a solid line, is congruent with the abscissa. The dotted lines define the threshold where the risk premium begins to rise above zero (lower dotted line) and where the risk premium is one (upper dotted line).

5.3 Aggregate productivity shock

For comparison reasons, the last row of Table 2 shows the results of a steady-state equilibrium with a 10% higher level of aggregate productivity. The results are as expected. An increase in productivity leads to a higher level of output, a slightly lower leverage ratio and a reduction in the minimum firm size. The wage rate rises and so does the number of firms. This implies a smaller average firm size which is in line with the observed increase in output.

6 Conclusion and outlook

This paper proposed a general equilibrium model that accounts for heterogeneous firms facing uninsurable idiosyncratic productivity shocks. A firm's capital consists of its own

enough funds to establish a new firm, since the minimum sustainable firm size is too large.

[23]The term "precautionary savings" is conventionally used for risk averse agents in incomplete markets. In this model, firms are risk neutral. Nevertheless, one could speak of precautionary motives due to the nature of costly defaults, as it is done for example in Valencia (2008).

funds (equity) as well as external funds (loans). However, due to the presence of financial frictions, namely, costly defaults, the possibility of accessing external funds is bounded by a rising risk premium. This premium can approach infinity for high leverage ratios which is equivalent to a borrowing constraint. This constraint prevents small firms from reaching the optimal production size.

The model permits analysis of the economy's supply side in more depth, as well as the interdependence between the size distribution of firms and economic aggregates. The analysis provides several insights. The equilibrium economy is populated with small and large firms in terms of equity. Less capitalized firms take on high loans in order to reach the optimal capital stock, whereas large firms do not. Accumulated retained earnings allow them to avoid the corresponding financing costs. Firms are also heterogeneous in equilibrium with respect to total capital. The less capitalized firms cannot reach the optimal capital stock through external funds due to the borrowing constraint. Dividends are only distributed if a firm has reached the optimal size in terms of equity.

The size distribution of firms is particularly interesting due to the decreasing returns to scale production function on a firm level. Many small firms produce more than fewer large firms with the same aggregate capital stock.

The paper also highlights the occurrence of multiple equilibria and their consequences. The "bad" equilibrium with high interest rates and high default probabilities effectively limits access to credit. This leads to lower leverage ratios and lower aggregate output. A comparatively negative effect on aggregate output also emerges when the volatility of idiosyncratic productivity shocks rises.

In comparison to the existing literature, I embedded the model of Cooley & Quadrini (2001) in a general equilibrium framework. In contrast to Cooley & Quadrini (2006), I keep risk premia and defaults for the most part endogenous. Both extensions, however, come at the cost of simplifying on other ends such as omitting the analysis of adjustment dynamics after aggregate shocks and a monetary authority as in Cooley & Quadrini (2006).

Further research could be pursued by extending the model to incorporate aggregate shocks and analyse impulse responses. However, this would increase the complexity significantly. Bounded rationality assumptions as proposed in Krusell & Smith (1998) would be needed. Also, the simplifying assumption of complementarity between labour and capital as input factors would have to be dropped.

7 Appendix

7.1 Computing the stationary equilibrium

This list highlights the main steps of the algorithm used to compute the stationary equilibrium described in Section 4.4.

1. Set $R = \frac{1-\beta}{\beta}$
2. Guess W.
3. Determine appropriate grids for the state variable $\vec{e} = [0, e_{\max}]$ and the two control variables $\vec{d} = [0, d_{\max}]$ and $\vec{l} = [0, l_{\max}]$ with exponentially rising distances between grid points.
4. Solve firm problem for R and W, i.e. determine policy functions $d^*(\tilde{e})$ and $l^*(e)$.
   (a) First calculate $\lambda(e, l)$ and $x(e, l)$ for each grid point combination using a grid for the idiosyncratic shocks $\vec{z} = [z_{\min}, z_{\max}]$.
   (b) Then use fast[24] value function iteration to determine optimal firm policy. Again, use $\vec{z}$ to compute the expected value function.
5. Determine optimal new firm size e^I and the corresponding return on investment R^I by using equations (23) and (22).
6. If $R^I > (<)R$, then increase (decrease) W using Bisection[25] and start again from step 2.
7. Determine the invariant, steady-state distribution of firms.
   (a) Start with an arbitrary, initial number of firms and a uniform distribution such that the aggregate capital stock equals $K = \gamma N$ and simulate.
   (b) In each period, K_t^{default} is lost due to default.
   (c) In each period, surviving firms grow by K_t^{growth}.
   (d) In each period, the number of new firms is computed using equation (26).
   (e) Continue until the number of firms does not change anymore.

[24] After five value function iterations use a first order Taylor approximation to project the steady-state value function from the two previous iterations.

[25] See Heer & Maussner (2004), Chapter 11.5.1

Chapter 3

Can relative consumption in incomplete markets explain the wealth distribution?

This paper investigates the effect of relative consumption in a heterogeneous agent, incomplete market economy. Households can only partially insure against idiosyncratic unemployment shocks. In contrast to standard models in this literature, households value not only their own consumption, but also relate it to the consumption level of the average household - a phenomenon also referred to as "keeping up with the Joneses". I discuss alternative utility function calibrations and find two results: In most scenarios, relative consumption leads to less inequality in wealth holdings. Poor households catch up with the average household by increasing precautionary savings as they are more afraid of falling behind. Rich households dissave faster because they gain twofold from excess consumption. Secondly, and also in contrast to standard models, redistributive tax policies have opposite welfare effects. The emerging deadweight losses in terms of capital accumulation, income and consumption, due to redistributive taxes, are overcompensated by the welfare gains derived from a more equal society. Thus, redistributive tax policies increase welfare.

Keywords: incomplete markets, heterogeneous agents, conspicuous consumption, keeping up with the Joneses, optimal taxation

JEL: E21, D52, D11, H21

1 Introduction

Do we only care about our own consumption or, also, about how our consumption level compares with others around us? If so, what are the economic consequences? Despite the fact that many bright minds have asked these questions for decades,[1] I try to contribute further insights to this field by embedding the phenomenon of relative consumption in a modern heterogeneous agent framework. In particular, I want to analyse the consequences of relative consumption behaviour on the shape of the wealth distribution and on optimal taxation.

As a motivational starting point consider the following observations: Suppose I have just bought a new, middle-class car. Would I then enjoy cruising around in the richest neighbourhood of the city? Probably not, because I would appreciate my new car much less, if everyone around me drove a *Ferrari*, a *Bentley* or a *Mercedes*. Alternatively, suppose I have just managed to successfully publish this paper in a top 50 journal. Would I be more or less happy if my colleague tells me right afterwards that he managed to publish an article in *Econometrica*?

One could come up with many such examples, which would generally lead to the same conclusion. Whether willingly or unwillingly, we inevitably compare ourselves to others. Even Adam Smith noted that women in England required better clothing when they appear in public than their counterparts in Scotland.[2] The message of this is clear. We do not only *care* about the consumption level of others but we also *act* accordingly. Thus, we often buy and consume goods that we would not buy and consume, if other people were around us.

For decades, many authors have found that relative consumption does play a role in our own well being and influences our decision making. Nevertheless, the vast majority of macroeconomic models ignore this fact and use the well established representative economic agent who only cares about his own level of consumption. This can become particularly worrying if conclusions about optimal taxation and welfare optimization are being derived. In this paper I would like to emphasize that misguiding conclusions can be drawn if models are restricted to agents without 'social' preferences.

This paper investigates the effects of relative consumption behaviour on the shape of the wealth distribution measured by the GINI coefficient. Utility functions reflecting this behaviour are embedded in an otherwise standard heterogeneous agent model with incomplete markets. Under plausible assumptions, relative consumption behaviour leads to a more equal society. This result is surprising and works against previous attempts to explain the observed inequality in wealth holdings with these types of models. The analysis also indicates that relative consumption behaviour is linked to heterogeneity of relative risk aversion in the utility function. In the second part of the paper, optimal taxation policies are discussed. A social planner could increase utilitarian welfare by implementing a redistributive tax policy. In a standard setting, that same tax reduces welfare because of the well-known disincentive effects on labour supply and capital accumulation.

The remaining part of this text is structured as follows. The next section gives a short

[1]See Veblen (1899), Duesenberry (1949), Easterlin (1957), Abel (1990), Harbaugh (1996), Arrow & Dasgupta (2009) and Section 2 of this paper.

[2]See Alpizar et al. (2005), page 1.

overview on related work. Section 3, outlines the model and shows how it can be solved. Section 4 demonstrates the results on the shape of the wealth distribution and other general equilibrium properties. Section 5 analyses optimal tax policies and 6 concludes.

2 Related work

There are three strands in the literature that have to be mentioned in order to understand the contribution of this paper: relative consumption, incomplete markets and optimal redistribution.

The theory of relative consumption is a large scientific field that spans multiple disciplines like psychology, sociology and economics. It goes as far back as Adam Smith, as mentioned above, or Veblen (1899) who give birth to the so called 'Veblen effect', i.e. rich people trying to achieve a greater status by buying and consuming luxury goods.

The term *relative consumption* is often also referred to as *consumption externalities*, *keeping up with the Joneses*, *conspicuous consumption*, *hedonic treadmill* or *habit formation*.[3] Of course, there are some fractional differences between these terms. Above all, consumption is not always the variable of interest. Also, the concepts differ in terms of the reference group.[4] However, all of them refer to the phenomenon of relativity in well-being.

This paper focuses on relative consumption with respect to other people and a vast amount of articles give evidence for this phenomenon. Easterlin (1957) observes that despite economic growth, both in terms of income and consumption, people nowadays are not happier than before and thereby gives birth to the so called *Easterlin Paradox*. He also observes that rich people are happier than poor people as long as one measures happiness within a particular country. However, poor people in rich countries tend not to be as happy as rich people in poor countries even if they are equally rich in absolute, real terms. In a more recent work, he reconfirms his findings and claims that "raising the income of all does not increase the happiness of all" (Easterlin 1995, p. 1). Many years earlier, Duesenberry (1949) had highlighted that people do not only care about their own consumption but compare themselves with their neighbours. This phenomenon is neither dependent on time nor geographic location. Half a decade later, Luttmer (2005) gives direct evidence for the relative consumption phenomenon in the U.S. by showing that people are less happy when their neighbours are richer. Kuhn et al. (2011) show that there is a significant, positive effect on consumption of neighbours of Dutch lottery winners. Clark & Oswald (1996) show that the satisfaction level with regard to income of 5000 British workers is inversely related to the income level of their reference group,

[3]See for example Abel (1990), Arrow & Dasgupta (2009), Easterlin (1957), Knight & Gunatilaka (2009), Bottan & Perez Truglia (2011), Keely (2005)

[4]The reference group can be other people, such as neighbours or the average consumer in a country, as it is the case in *keeping up with the Joneses* or (external) *habit theory*. Alternatively, the reference group can be the own past self as in *hedonic treadmill* models or (internal) *habit theory*.

i.e. workers in the same industry with a similar educational background. Charles et al. (2009) observe relative consumption effects but also show that they decline with the average income of the reference group.[5]

It should be noted that this paper does not directly contribute to the literature on relative consumption. Rather, I take these findings as given and analyse the consequences from a new perspective by embedding relative consumption in a dynamic, heterogeneous agent, general equilibrium framework.

Many authors have used relative consumption behaviour to explain the asset premium puzzle.[6] Some others have used it to find welfare optimizing tax policies. Oswald (1983), for example, analyses optimal taxation in a Mirrlees economy enriched by relative consumption preferences.[7]

Ljungqvist & Uhlig (2000) demonstrate that overconsumption effects caused by relative consumption increase the amplitude of business cycles and recommend the application of appropriate tax policies. Stracca & al Nowaihi (2005) show that, due to relative consumption, multiple equilibria in the consumption-savings decision can occur. They apply results from prospect theory[8] to the field of relative consumption: their agents have convex utility below the level of consumption of the average consumer and standard, concave utility above that level.

Arrow & Dasgupta (2009) prove for a broad range of utility functions that in a complete market economy relative consumption leads to inefficient outcomes as long as labour is endogenously supplied. They show that agents work, save and consume more than they would do, if they did not care about the consumption level of the average consumer. But since they use a representative agent framework, they cannot state anything about the distributional effect nor about redistributive tax policies.

This, however, is a popular field in economics: Aiyagari (1994), Krusell & Smith (1998), Heer & Trede (2003), Castaneda et al. (2003) and Davila et al. (2005) have tried to calibrate extensions of so called Bewley (1977) models in order to match observed wealth and income distributions.[9] Doing so, however, is particularly difficult and has so far only partially been accomplished. Heer & Trede (2003), for example, manage to achieve a GINI coefficient with respect to wealth of 0.4 for Germany, whereas empirical

[5]The amount of evidence for relative consumption effects is almost infinite. More outstanding evidence can be found in Senik (2005), Christen & Morgan (2005) and Alpizar et al. (2005).

[6]See for example Abel (1990), Gali (1994) or Campbell & Cochrane (1999).

[7]In contrast to this paper, the model introduced by Mirrlees (1976) is, among other things, static and limited to a partial equilibrium economy.

[8]See Tversky & Kahneman (1991)

[9]Bewley (1977) introduced models with heterogeneous agents that are subject to idiosyncratic income shocks. The shocks are only partially insurable, thus, markets are incomplete in the spirit of Arrow & Debreu (1954). These so called Bewley models were then embedded in a dynamic general equilibrium framework by Huggett (1993) and Aiyagari (1994). Another milestone was accomplished when they were enriched by aggregate productivity shocks by Krusell & Smith (1998) - a feature that this paper, however, abstains from. Most of these authors also discuss optimal tax policies. Most recent and prominent is the work by Conesa et al. (2009) who use an Aiyagari (1994) model enriched by overlapping generations to analyse optimal taxation.

estimates are well above 0.6. Krusell & Smith (1998) only match the observed wealth distribution in the U.S. if they use heterogeneous discount rates. Castaneda et al. (2003) calibrate until the most productive worker, among four different types, is 1061 times more productive than the least productive worker.

García-Peñalosa & Turnovsky (2008) are among the few authors who analyse the wealth distribution in a relative consumption environment. In their model, agents are heterogeneous in terms of their initial wealth endowment and their reference point. These assumptions are necessary to generate wealth inequality since they abstain from market incompleteness and idiosyncratic income shocks. They find that relative consumption leads to less inequality in terms of a faster convergence towards the steady-state distribution. Ordabayeva & Chandon (2011) invert the question by analysing experimentally whether exogenously decreasing inequality leads to lower consumption and thus to higher savings. They find that the results go into both directions.

To the best of my knowledge, no one has analysed the wealth distribution and optimal taxation with respect to relative consumption in an otherwise standard heterogeneous agent model with incomplete markets. Given the vast amount of evidence for relative consumption, the standard models might omit an important aspect. Therefore, combining both strands seems to be overdue and necessary in order to improve fiscal policy recommendations.

3 Model

This section outlines the economic setting. The model is based on the standard heterogeneous agent, incomplete market framework proposed by Aiyagari (1994) and is enriched by agents who care about relative consumption.

3.1 Households

There exists an infinite number of infinitely lived households j of mass one, i.e. $j \in [0,1]$. Each household is endowed with one unit of time per period and maximizes the discounted sum of future instantaneous utilities. At $t = 0$ a household's intertemporal, lifetime utility is given by:

$$v_0 = \max \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u(c_t, C_t, 1 - n_t)\right] \tag{1}$$

where c_t denotes the household's own consumption, C_t is the average consumption per household, n_t is the amount of labour supplied and β is the discount factor.[10] Hence, the

[10]Throughout this text, upper case characters denote aggregates, whereas lower case characters denote individual, household values. Thereby, an index j to denote an individual household is skipped for the sake of notational simplicity. Note that due to the unit mass of households C_t refers to both, aggregate and average consumption.

household values not only its own consumption of goods and leisure but relates the consumption of goods to the average household.[11] The functional form of the instantaneous utility function u features the standard concavity properties with respect to individual consumption and labour supply:[12]

$$u_c > 0, u_{cc} < 0, u_n < 0, u_{nn} < 0 \tag{2}$$

As suggested in the literature of relative consumption,[13] individual utility decreases when average consumption increases, thus:

$$u_C < 0 \tag{3}$$

Additionally, most authors require that individual marginal utility increases with average consumption:[14]

$$u_{cC} > 0 \tag{4}$$

Dupor & Liu (2003) disentangle both effects and call the first one *jealousy* and the latter *keeping up with the Joneses.* Most other studies use both effects collectively.

Each household can accumulate capital k_t by saving a share of its income.[15] r_t denotes the net interest earned on capital and w_t denotes the wage rate. Households face partially insurable, idiosyncratic employment shocks that are modelled by a first order Markov chain.[16] Let $e_t \in \{0, 1\}$ denote the current employment state, where 1 stands for employed and 0 for unemployed. The conditional probability of changing that state, $\mathcal{P}(e_{t+1}|e_t)$, is given by the transition matrix

$$P = \begin{bmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{bmatrix} \tag{5}$$

where, for example, ρ_{01} denotes the probability of switching from unemployment to employment. Note that the rows in this matrix add up to one. The government runs an unemployment insurance system. If a household is unemployed, it earns unemployment benefits b_t. Therefore, households have to pay taxes x_t on their total income, i.e. labour

[11]Models with aggregate shocks distinguish between the *catching up with the Joneses* and *keeping up with the Joneses* effect. In the first case, households value average consumption in the previous period, whereas in the latter case, they value current average consumption. Since only stationary equilibria without aggregate shocks are considered here, the distinction becomes irrelevant. The text therefore sticks to the notationally simpler version of current average consumption.

[12]u_c, u_C and u_n denote partial derivatives of u with respect to c, C and n

[13]See Abel (1990), Campbell & Cochrane (1999), Ljungqvist & Uhlig (2000) and Arrow & Dasgupta (2009)

[14]Arrow & Dasgupta (2009) impose the slightly stronger assumption of u being concave when c and C increase together.

[15]Throughout this text capital, wealth and assets are used interchangeably as there is just one good in this economy.

[16]Markov chains are discrete valued Markov processes. They are an excellent approximation of autoregressive processes - a feature that holds empirically for the observed process of switching between employment and unemployment. In order to find the probability values of the transition matrix that match a given autoregressive process, see Tauchen (1986).

and capital income are both taxed at a rate τ.[17] Households can self-insure against employment shocks by saving and borrowing assets. However, the household cannot borrow indefinitely. The borrowing constraint is set to:[18]

$$k_t \geq 0 \tag{6}$$

Based on these budget constraints the law of motion for the individual household's capital stock is

$$k_{t+1} = (1 + r_t)k_t + e_t w_t n_t + (1 - e_t)b_t - c_t - x_t \geq 0 \tag{7}$$

where $x_t = \tau(r_t k_t + e_t w_t n_t)$. If a household is unemployed, it consumes all time endowment as leisure, hence, $e_t = 0 \Rightarrow n_t = 0$. This assumption could be altered without changing the results significantly.

Households are heterogeneous not only with respect to employment status but also wealth i.e. the amount of capital a household owns. Let $f_t(e, k)$ denote the two-dimensional distribution of households with respect to these two state variables. Also, each household has two control variables: the amount of consumption $c(e, k)$ and the amount of labour supply $n(k)$. Since a household does not supply any labour while it is unemployed, the labour supply decision does not depend on the employment status.

Given the current distribution f_t, the aggregate capital stock K_t, aggregate labour supply N_t and aggregate consumption C_t can be computed as follows:

$$K_t = \sum_{e \in 0,1} \int_0^\infty k f_t(e, k) dk \tag{8}$$

$$N_t = \int_0^\infty n(k) f_t(1, k) dk \tag{9}$$

$$C_t = \sum_{e \in 0,1} \int_0^\infty c(e, k) f_t(e, k) dk \tag{10}$$

Each of these values also denotes average values since the mass of households is normalized to one. Employment is given by

$$E_t = \int_0^\infty f_t(1, k) dk \tag{11}$$

and unemployment equals $1 - E_t$.

All households act rationally and build their expectations likewise. Optimal decisions of the households with respect to consumption and labour supply are denoted by $c^*(e, k)$

[17] In Section 5, a more sophisticated tax system will be discussed that, among other things, distinguishes between capital and labour taxation.

[18] This constraint could also be set to negative values without changing the results qualitatively. The smallest possible value would be the 'natural' borrowing limit highlighted in Aiyagari (1994) which is the discounted sum of all future income flows in a worst case scenario, here, $\frac{b}{r}$, where b and r are the steady state minimum income and interest rate, respectively.

and $n^*(k)$, respectively. Households take average consumption C as given.

3.2 Production

Firms are competitive and homogeneous and take factor prices w_t and r_t as given. The production technology of the representative firm has a standard Cobb-Douglas form:

$$Y_t = K_t^\alpha N_t^{1-\alpha} \tag{12}$$

There are no aggregate productivity shocks in this economy. Market clearing wages and (net) interest rates equal their respective marginal products

$$w_t = (1-\alpha)\left(\frac{K_t}{N_t}\right)^\alpha \tag{13}$$

and

$$r_t = \alpha\left(\frac{N_t}{K_t}\right)^{1-\alpha} - \delta \tag{14}$$

where δ denotes the depreciation rate of capital per period.

3.3 Government

The government collects taxes X_t and distributes unemployment benefits B_t. The latter is simply the product of the mass of unemployed households times the individual unemployment benefit b_t

$$B_t = b_t(1 - E_t) \tag{15}$$

whereas the amount of taxes collected by the government is the sum of all labour income taxes and all capital income taxes.

$$X_t = \sum_{e \in 0,1} \int_0^\infty \underbrace{\tau(r_t k + e w_t n(k))}_{x_t(e,k)} f_t(e,k) dk \tag{16}$$

Section 5 will involve a discussion of optimal tax policies. For this matter, it is necessary to define a measure that determines aggregate welfare. Let $v(e_t, k_t) = v_t$ be the present value of all discounted future instantaneous utilities of a household as defined in (1) at time t, given the current state (e_t, k_t). The goal of the government is to maximize aggregate, utilitarian welfare given by

$$V_t = \sum_{e \in 0,1} \int_0^\infty v(e,k) f_t(e,k) dk \tag{17}$$

Obviously, this is not a trivial task, since there is a trade-off between individual and average consumption. Higher taxes increase disincentives to accumulate capital and supply labour. Thereby, aggregate production, income and consumption will be reduced. On the other hand, increasing taxes influences the shape of the wealth distribution and thereby consumption behaviour, too. Since agents gain their utility also from relative consumption, a more equal wealth distribution, as well as a lower average consumption, result in a welfare gain.

3.4 Stationary equilibrium

Since there are no aggregate shocks, the distribution $f(e,k)$ of households with respect to employment and capital is stationary in equilibrium. Also, the aggregate capital stock K, labour supply N, employment E and welfare V are constant. Consequently, factor prices w and r as well as aggregate and average consumption per household C do not change over time. No laws of motion with respect to the state variables of the economy are needed. As a consequence, time indices are dropped in what follows.

For a computational solution, it is necessary to convert the problem of the household to a recursive form. Given w, r and C and assuming the usual No-Ponzi-Condition $\lim_{t\to\infty} k_t = 0$, the household's optimization problem can be formulated recursively:[19]

$$v(e,k) = \max_{c,n} u(c, C, 1-n) + \beta \mathbb{E}\left[v(e', k')\right] \tag{18}$$

subject to

$$k' = (1+r)k + ewn + (1-e)b - c - x \geq 0 \tag{19}$$

where e evolves according to the transition matrix P and $x = \tau(rk + ewn)$.

The stationary, recursive equilibrium for a given tax policy τ is then given by the value function $v(e,k)$, the corresponding optimal consumption and labour supply functions $c^*(e,k)$ and $n^*(k)$, a stationary distribution $f(e,k)$ of households, the aggregates K, N, C, E, Y and the factor prices w and r as defined in (8) to (14) such that the government budget is balanced:

$$B = X \tag{20}$$

and the goods market clears:

$$Y - \delta K = C \tag{21}$$

Unemployment benefits b are endogenously determined in equilibrium by the exogenous tax rate τ through equations (15), (16) and (20).

3.5 Solution algorithm

This section briefly illustrates how the previously defined stationary equilibrium can be computed. For a detailed description see the Appendix 7.1. The algorithm comprises the

[19] As time indices are dropped, an apostrophe is used to denote the value of a variable in the subsequent period.

following steps:

1. Compute the ergodic distribution of employed households E from the transition matrix P.

2. Guess the aggregate capital stock K and labour supply N.

3. Compute the net interest rate r, the wage rate w, the unemployment benefits b and aggregate (average) consumption C.

4. Solve the problem of the household as defined by (18) and (19).

5. Simulate a large number of households given $c^*(e, k)$ and $n^*(k)$ and thereby determine the steady-state distribution of assets $f(e, k)$.

6. Calculate K and N from $f(e, k)$ and compare with the initial guesses. Update and repeat steps 2 to 6 if guesses and outcome do not match.

The first step can easily be accomplished by calculating the time-invariant probability distribution of the given first-order Markov Chain. The fourth step of the algorithm, i.e. solving the problem of the household, is accomplished by value function iteration.[20] The intratemporal labour supply decision can be solved analytically, if the instantaneous utility function is additively separable with respect to labour and consumption. Hence, for any value of consumption c, there is an analytical solution for the optimal labour supply $n^*(c)$. This term can be found by determining the first order conditions with respect to c and n of (18) after setting $e = 1$ and substituting k' with (19). As a result, one gets[21]

$$u_c + \beta v_k \cdot (-1) = 0 \tag{22}$$

$$u_n + \beta v_k \cdot (1 - \tau) w = 0 \tag{23}$$

which simplifies to

$$u_n + u_c (1 - \tau) w = 0 \tag{24}$$

If u is additively separable with respect to leisure and consumption, then the last equation can be solved for the optimal labour supply. This decreases the computational complexity of the problem significantly.

An interesting partial equilibrium result can be derived from this equation: If consumption of the reference group, i.e. average consumption, rises, everybody will increase labour supply. Using the calculus of implicit functions, it holds that $\frac{\partial n}{\partial C} = -\frac{u_{cC}(1-\tau)w}{u_{nn}}$ which is larger than zero given conditions (2) and (4), i.e. if marginal utility of consumption rises with average consumption and if disutility of labour is strictly convex.

[20] General proofs of existence and convergence of the procedure are given in Stokey et al. (2009) as well as in Huggett (1993)

[21] u_c, u_n and v_k denote partial derivatives of u and v with respect to c, n and k

3.6 Calibration

Three different functional forms of the utility function are discussed. The first two have already been used in the literature, while the third one is an original contribution. All three are additively separable with respect to leisure and consumption.[22] They mainly differ in the ways that individual and average consumption are valued.

As a benchmark, I use a functional form based on Abel (1990) and extended by endogenous labour supply.[23]

$$u^a(c_t, C_t, 1-n_t) = \frac{\left(\frac{c_t}{C_t^\gamma}\right)^{1-\eta}}{1-\eta} + \varphi_0 \frac{(1-n_t)^{1-\varphi_1}}{1-\varphi_1} \tag{25}$$

The parameters $\varphi_0 \geq 0$ and $\varphi_1 \geq 0$ influence the average labour supply as well as the coefficient of variation of labour supply. $\eta \geq 0$ equals the coefficient of relative risk aversion and $\gamma \geq 0$ determines the importance of relative consumption. If $\gamma = 0$ there is no spillover effect from the average consumer on a household's own utility. The utility function satisfies the assumptions, (2), (3) and (4), given above.

A second functional form is taken from Ljungqvist & Uhlig (2000) in which the relative consumption effect enters the consumption term additively instead of multiplicatively.

$$u^l(c_t, C_t, 1-n_t) = \frac{(c_t - \gamma C_t)^{1-\eta}}{1-\eta} + \varphi_0 \frac{(1-n_t)^{1-\varphi_1}}{1-\varphi_1} \tag{26}$$

This function also satisfies the assumptions (2), (3) and (4). Moreover, it has the interesting effect of a non-constant coefficient of relative risk aversion, namely $\frac{-cu_{cc}}{u_c} = \eta \frac{c}{c-\gamma C}$. Here, η is the lower bound of relative risk aversion which is asymptotically approached for $c \to \infty$. A disadvantage of this functional form is the strictly positive lower bound of consumption $c > \gamma C$. Also, γ is bounded from above by $\gamma < 1$. If γ was one, the lower bound of consumption would have to be equal to the average level of consumption, which is not possible. Also note that the coefficient of relative risk aversion goes to infinity, if consumption approaches the lower bound, γC, from above.[24]

The third functional form, I consider, has an additive relative consumption effect outside the consumption term. This leads to slightly different properties compared to

[22] Additively separable utility functions, in contrast to multiplicative Cobb-Douglas types, are frequently used in computational models (see Heer & Trede 2003 or Conesa et al. 2009) due to the computational simplifications discussed in the previous section.

[23] The utility function used by Abel captures both, relative consumption with respect to the average consumer as well as to the own past consumption (internal habits). As this model only focuses on the first effect, the respective parameter is set to $D = 0$ in the utility function of Abel. Thereby, the effect of a consumer's own past consumption does not play a role anymore. Also, due to the analysis of stationary equilibria, it is irrelevant whether C_t or C_{t-1}, as proposed by Abel, enters the utility function.

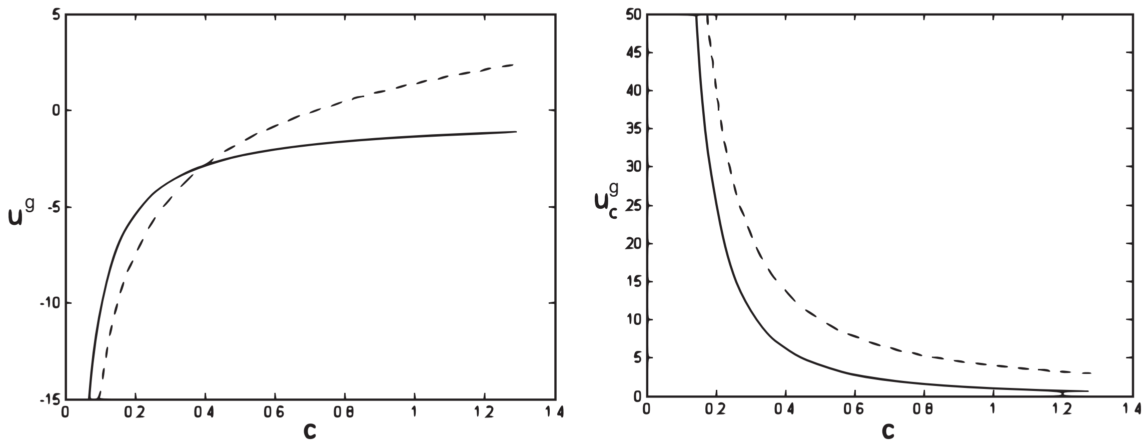
[24] The previous two utility functions have been used in several studies. However, there is no clear argument which one captures relative consumption behaviour better. Alpizar et al. (2005) analyse both utility functions in an experiment without giving a hint on which one of them fits best. Clark & Oswald (1998) discuss theoretical differences between the two functions.

the previous two functions:

$$u^g(c_t, C_t, 1-n_t) = \gamma \log\left(\frac{c_t}{C_t}\right) + \frac{c_t^{1-\eta}}{1-\eta} + \varphi_0 \frac{(1-n_t)^{1-\varphi_1}}{1-\varphi_1} \tag{27}$$

The previous two functions simply shift utility downwards, hence, making everybody worse off if $\gamma > 0$, that is, if relative consumption behaviour is present. The idea behind this function is somewhat different. Households above the average consumer receive an additional felicity surplus whereas those below the average consumer get a deduction. Figure 1 highlights this fact and also shows the change in marginal utility once relative consumption plays a role. In contrast to Ljungqvist & Uhlig, this function is defined for all values of $c > 0$. Still, it features a variable coefficient of relative risk aversion, $\frac{\gamma+\eta c^{1-\eta}}{\gamma+c^{1-\eta}}$, but with slightly different properties: it converges to η from below for $c \to 0$ and to 1 for $c \to \infty$ if $\eta > 1$. Moreover, relative risk aversion does not rise with aggregate consumption C.

However, the function only satisfies conditions (2) and (3) but fails on condition (4): marginal utility does *not* rise when average consumption rises. Using the words of Dupor & Liu (2003), the function expresses *jealousy* but not the phenomenon of *keeping up with the Joneses*.



(a) Utility. The intersection of both lines is at $c = C$.

(b) Marginal utility.

Figure 1: Properties of the proposed utility function u^g for values of $\gamma = 0$ (solid line) and $\gamma = 3.0$ (dashed line).

The decreasing coefficient of relative risk aversion in the latter two utility functions is particularly interesting since it reflects a typical finding in the data, that is, the share of risky assets increases with total wealth.[25] It should be noted, however, that this debate is highly controversial and ongoing as highlighted for example in Guiso & Paiella (2008), Brunnermeier & Nagel (2008) and Chiappori & Paiella (2011). The only common agreement is that risk aversion is very heterogeneously distributed. Another frequent

[25] See, for example, Cohn et al. (1975), Levy (1994) or Ogaki & Zhang (2001)

finding is that liquidity constrained agents or agents close to the subsistence level show very high levels of risk aversion. In general, however, the heterogeneity in risk aversion is only partially and sometimes even not at all explicable by wealth differences.

All three utility functions, u^a, u^l and u^g collapse to the same functional form, if relative consumption is irrelevant, i.e. if $\gamma = 0$:

$$u(c_t, C_t, 1 - n_t) = \frac{c_t^{1-\eta}}{1-\eta} + \varphi_0 \frac{(1 - n_t)^{1-\varphi_1}}{1 - \varphi_1} \tag{28}$$

The calibrated model uses exogenous parameters taken from Heer & Trede (2003) and Heer & Maussner (2004) which match the German economy.[26] The parameters are shown in Table 1. The values of the discount rate, the capital share, relative risk aversion and depreciation have become standard in the literature. The unemployment probabilities imply an average unemployment rate of 8%. The weight on the disutility of labour φ_0 is calibrated in order to match an average labour supply of 30%. φ_1 is held constant throughout the experiments. Therefore, the coefficient of variation in total labour supply does change, but only at a very small and economically insignificant scale. The tax rates are chosen in order to match an unemployment benefit of approximately fifty percent of the average labour income.

The utility function parameters η and φ_0 that affect relative risk aversion and average labour supply are relevant for the benchmark utility function u^a, only. When employing the utility functions u^l and u^g, these parameters are calibrated in order to match the same outcomes in terms of average relative risk aversion and labour supply as in the benchmark setting.

In comparison to Heer & Trede (2003) some simplifications have been made. All households have the same labour productivity and government consumption is zero. The latter leads to lower taxes whereas constant labour productivity leads to a relatively moderate GINI coefficient. However, these simplifications do not change the main results of the paper which are the qualitative effects of relative consumption on the shape of the wealth distribution, as well as on optimal taxation policies. The main message of the paper is that a small, but plausible and empirically supported, modification of the utility function leads to entirely different optimal tax policies. Thus, as long as the literature does not yet concordantly agree on the preferences of individuals, policy recommendations based on calibrated micro-founded models might be misleading. This conclusion is independent of the type of calibration.

[26]Their values are selected in order to match data from the Socio-Economic Panel for Germany (GSOEP), the German Institute for Economic Research (DIW) and the German Statistical Office.

Parameter	Value	Description
β	0.96	discount rate
α	0.36	capital share
δ	0.04	depreciation rate
ρ_{11}	0.9565	probability to stay employed ($\rho_{10} = 1 - \rho_{11}$)
ρ_{00}	0.5	probability to stay unemployed ($\rho_{01} = 1 - \rho_{00}$)
η	2	relative risk aversion in benchmark setting
φ_0	0.13	labour supply parameter in benchmark setting
φ_1	10	labour supply parameter
τ	0.01	tax in benchmark setting

Table 1: Numerical parameters

4 Results

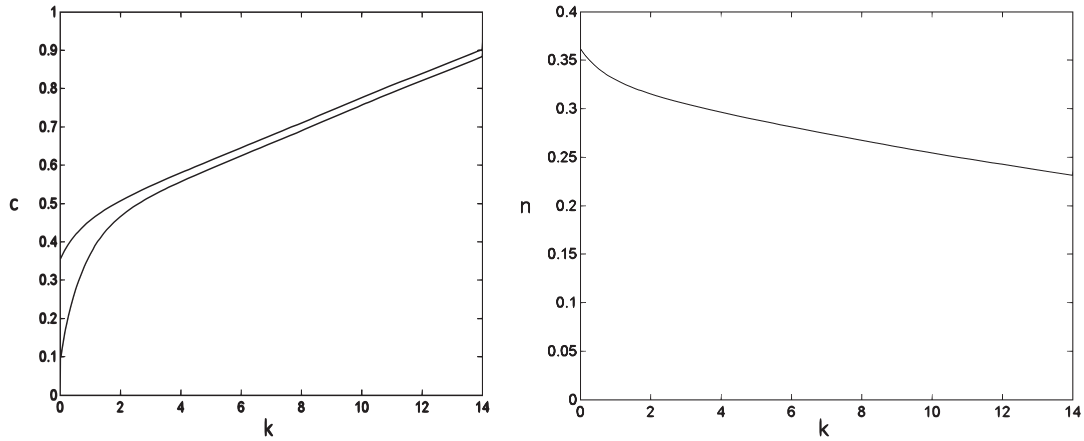
This section describes the findings on how relative consumption affects the steady-state of an economy. Does a general equilibrium model in which individual, heterogeneous households care about the consumption of others lead to more or less inequality in wealth holdings in comparison to standard models? Optimal taxation policies in a relative consumption environment will be discussed in the next section.

Of course, the analysis in this section has to be pursued with care since economic settings with different preferences are compared. An economy in which relative consumption behaviour is present can be described by either one of the three proposed utility functions u^a, u^l and u^g with the respective parameter γ set to values greater than zero. Clearly, once preferences are changed, outcomes are in general not comparable. In order to answer the question of whether relative consumption increases or decreases inequality, I calibrate each utility setting to the same observed economic descriptives e.g. aggregate labour supply and average relative risk aversion.

The results are compared to the benchmark setting with $\gamma = 0$, i.e. where relative consumption does not play a role. Figures 2 to 4 display the results of the benchmark setting qualitatively.

Figure 2 shows the optimal policy functions of employed and unemployed households $c^*(e, k)$ and $n^*(k)$. As usual in incomplete market models, consumption drops significantly for low levels of wealth. This is due to the fact that agents want to build up a capital stock buffer or precautionary savings. Therefore, they reduce their consumption rate in this range. Also, note the nonlinear labour supply: being poor leads to higher work effort in order to build up precautionary savings.

Figure 3 shows the value functions of employed and unemployed households. Similar to the consumption functions, they are almost identical for high levels of capital stock. A high stock of precautionary savings serves as an insurance against unemployment. Thus, an unemployed household does not change its consumption behaviour significantly in this area and therefore has almost the same lifetime utility as the employed household. Figures 4(a) and 4(b) show the distribution of wealth and the Lorenz curve in the stationary equilibrium.[27]



(a) Optimal consumption of employed (upper line) and unemployed (lower line) households.

(b) Optimal labour supply of unemployed households.

Figure 2: Policy functions in the benchmark setting.

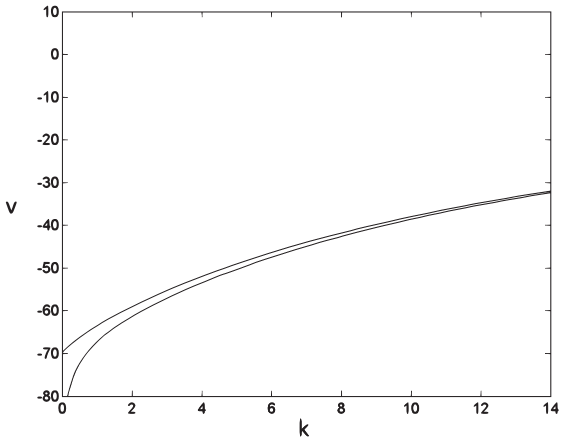
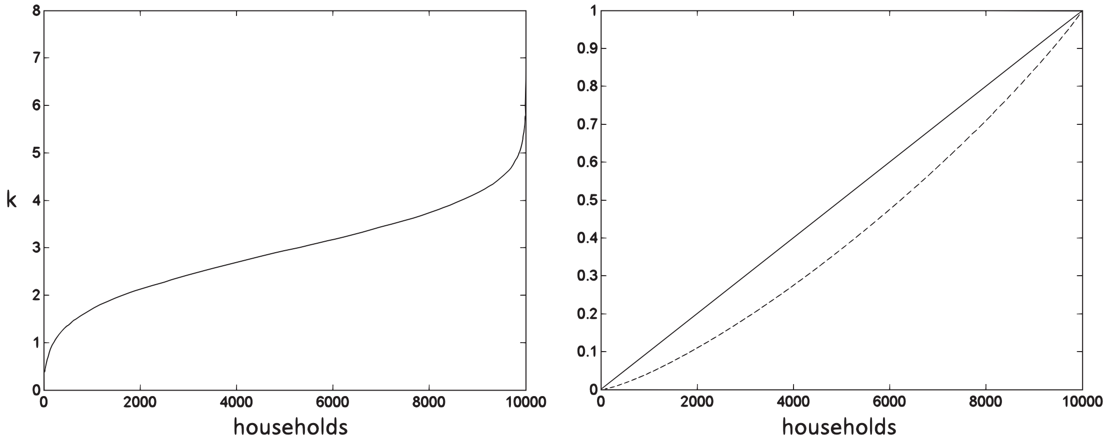


Figure 3: Value function of employed (upper line) and unemployed (lower line) households in the benchmark setting.

Table 2 shows the results of all computational experiments discussed in this section. The first row displays the results of the benchmark setting in which relative consumption is irrelevant, i.e. in which $\gamma = 0$. All three utility functions are equivalent in that case. This serves as the reference point to which the other results are compared to.

Setting 2 shows the result when the utility function u^a of Abel (1990) is implemented. γ is set to one, which is the same value that was used in his paper. Trivially, this results in exactly the same outcome. Agents have no influence on average consumption. Hence,

[27]The GINI coefficient of 0.18 does not match the data, that shows values well above 0.5 in all developed economies (Davies et al. 2009), since all households are equally productive in this model. As in Aiyagari (1994), Heer & Trede (2003) and others, this could easily be adjusted by introducing an exogenous labour productivity process. However, while making the model more complicated, it would not help answering the research question of this paper, namely, whether relative consumption increases or decreases wealth inequality.



(a) Distribution of assets. (b) Lorenz curve.

Figure 4: Outcome in the benchmark setting. The GINI coefficient is 0.18.

Setting	Parameters				Results				
	utility	γ	η	φ_0	$\overline{RRA}$	N	K	C	GINI
1	$u^{a,l,g}$	0.00	2.00	0.13	2.00	100%	100%	100%	100%
2	u^a	1.00	2.00	0.07	2.00	100%	100%	100%	100%
3	u^l	0.10	2.00	0.16	> 2	100%	101%	100%	89%
4	u^l	0.10	1.80	0.15	2.00	100%	100%	100%	93%
5	u^g	3.00	2.00	0.35	< 2	100%	99%	99%	108%
6	u^g	3.00	3.00	0.45	2.07	100%	101%	100%	87%

Table 2: Calibration results of different economic settings in comparison to the benchmark setting shown in the first row. $\overline{RRA}$ denotes the average coefficient of relative risk aversion. The last four columns show the relative size of the aggregates N, K and C as well as the GINI coefficient with respect to the first row.

the consumption term of their utility function is simply multiplied by the factor $C^{-\gamma(1-\eta)}$ which is smaller than one if aggregate consumption C is smaller than one and vice versa. Consequently the weight of the consumption term in the utility function is changed which results in a different labour supply, ceteris paribus. If the model is calibrated for the same aggregate labour supply, naturally, the outcome is the same again. To be precise, if $\gamma = 1$, the weight of labour supply $\varphi_{0,\gamma=1}$ that matches the same aggregate labour supply as before, has to be

$$\varphi_{0,\gamma=1} = C\varphi_0^\eta \tag{29}$$

where φ_0 is the weight of labour supply in the benchmark setting with $\gamma = 0$.

Analogous results are obtained, if labour supply is inelastic and set to $n = N/E$ for the employed household. Switching to relative consumption does not change the outcome. This is due to the fact that the utility function is simply shifted vertically in that case.

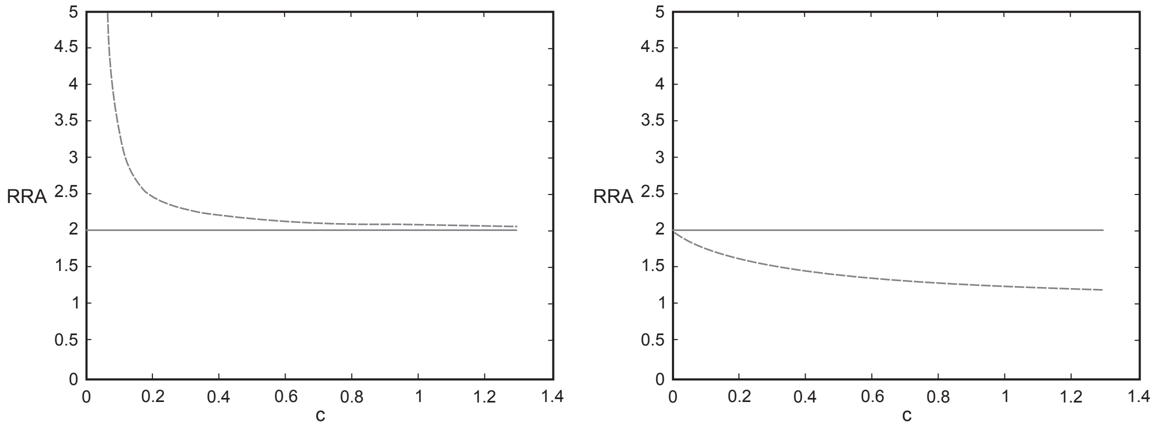
Setting 3 applies the utility function u^l used by Ljungqvist & Uhlig (2000).[28] Once this setting is calibrated via φ_0 to match the same labour supply as in the benchmark setting, the distribution of wealth measured by the GINI coefficient drops significantly by 11 percentage points. Hence, the relative consumption effect that emerges when using u^l instead of u^a results in a more equal economy. The reason for this phenomenon becomes clear once we analyse the already mentioned heterogeneity of the coefficient of relative risk aversion, henceforth RRA. As can be seen in Figure 5(a), RRA explodes for households whose consumption level is very low, thus, for very poor households. Consequently, poor agents increase their amount of precautionary savings comparatively more than rich agents, which results in more equality in wealth holdings. From an analytical perspective, relative consumption can thus be linked to the corresponding increase in RRA.

Setting 4 shows what happens to an economy that is once more calibrated to match the same average RRA as the benchmark model.[29] Hence, the calibration of η shifts the graph in 5(a) downwards, thus making everyone less risk averse. Not surprisingly, inequality rises in comparison to setting 3 but still stays well below the benchmark setting. Even though rich people are now less risk averse than before, and would thereby be confident with lower values of precautionary savings, poor people overcompensate for this phenomenon. This is not only due to the fact that RRA in this area is asymmetrically higher but that it is also more relevant for people closer to the liquidity constraint.

The last two settings, 5 and 6, repeat the analysis for the utility function u^g. Setting 5, leads to more inequality in wealth holdings or a higher GINI coefficient than the

[28] Note that the value of γ cannot be compared between the different utility functions. For example, the quantitative effect is much higher in the utility function of Ljungqvist & Uhlig than in the utility function of Abel. Hence, the analysis in this section takes the variable γ as an on-off variable. Either there is relative consumption ($\gamma > 0$) or there is not ($\gamma = 0$). In an experiment among students at the University of Costa Rica, Alpizar et al. (2005) estimate the average value of γ to be between 0.50 and 0.75 in the utility function of Abel and between 0.46 and 0.66 in the function of Ljungqvist & Uhlig. However, the experiment did not involve any risk and the estimated utility functions used $\eta = 0$.

[29] See Appendix 7.2 for details on the calibration procedure.



(a) Relative risk aversion of utility function u^l for $\gamma = 0.1$ (dashed).

(b) Relative risk aversion of utility function u^g for $\gamma = 1$ (dashed).

Figure 5: Coefficients of relative risk aversion with respect to different consumption levels and utility specifications. The solid line plots the coefficient of relative risk aversion of utility function u^a which is constant and equal to η for all values of γ.

benchmark setting. Without calibrating for the same average RRA, all agents are less risk averse than in the benchmark setting, as Figure 5(b) shows. Again, calibrating for the same average RRA results in a lower GINI coefficient, for the same reasons as given above. In that sense, the results provided here are in line with the ones found by García-Peñalosa & Turnovsky (2008), however, in a completely different but probably more realistic economic setting with stochastic income shocks, borrowing constraints and therefore endogenous heterogeneity in wealth holdings.

Can relative consumption in incomplete markets explain the wealth distribution? Probably not. Nevertheless, this result is still an interesting one in the sense that it challenges the existing literature even more. As mentioned in Section 2, the research community still tries hard to explain the observed inequality in wealth holdings. As shown here, introducing relative consumption works further against them. Therefore, previous arguments put forward to explain the wealth distribution, such as stochastic discount factors or heterogeneous labour productivity, have to be even more pronounced or an entirely new set of arguments has to be found.

As a final remark, I would like to address the question of whether there might be another utility function specification that causes increasing inequality apart from setting 5. An attempt worth mentioning is that of Stracca & al Nowaihi (2005) who use an S-shaped utility function in the spirit of Tversky & Kahneman (1991). Their reference point, or turning point of marginal utility, is the consumption level of the average consumer. Above that threshold, the utility function is, as usual, concave. Below, however, it is convex. In a complete market, partial equilibrium with a representative consumer they find that an infinite number of multiple equilibria emerges.[30] They point out that this

[30] Bogliacino & Ortoleva (2011) analyse a similar model and conclude that two equilibria emerge: one in which all wealth is equally distributed and one in which wealth is highly polarized between a rich and a poor class.

finding opens the discussion on how an entirely new set of factors, such as advertising and social norms, might determine aggregate outcome.

It would be interesting to observe the effect of such a utility specification on the incomplete market, general equilibrium model of this paper. Given the arguments outlined above, this would probably lead to significantly increasing inequality in wealth holdings. Households would increase their consumption level once they are below the average consumer and abstain from building up precautionary savings. From today's perspective, it is, however, not very convincing that poor agents are risk loving and the average agent is risk neutral. As mentioned above, most of the empirical analysis points in the opposite direction.[31]

5 Optimization

This section analyses welfare optimizing tax policies in an economy with households that care about relative consumption. The analysis follows the standard approach of tax optimization as conducted, for example, in Conesa et al. (2009): starting from a given calibrated equilibrium, the welfare effects of different tax policies are analysed and the optimum is determined. Moreover, emerging differences in comparison to standard utility functions are emphasized. As it turns out, a welfare increasing tax policy in a standard setting might not have the same effect in a relative consumption setting.

As mentioned in Conesa et al. (2009), finding the optimal tax policy is, in general, an infeasible task. This would require optimizing welfare over the infinitely dimensional functional space of all possible tax functions for income and consumption. To cope with this problem the literature typically restricts possible tax policies to a given parametric function and optimizes over the parameter set.

During this analysis, I restrict the tax policy function to a progressive tax system, since it is sufficient to show the main argument. Technically, I allow the government to implement a redistributive, balanced tax system on top of the already existing unemployment insurance system. Therefore, the government raises another flat tax ν_n on labour and ν_k on capital income and redistributes the collections via lump sum transfers d. In order to conduct a thorough analysis, I also split the tax rate τ, which has been used for the unemployment insurance in the baseline model, into τ_k and τ_n. Thus, the new tax function of a household reads as follows:

$$x = (\tau_k + \nu_k)rk + e(\tau_n + \nu_n)wn - d \tag{30}$$

[31] A first idea of how labour supply reacts to such a utility function in this economic setting can be obtained by observing the optimal labour supply condition in (35) in the Appendix 7.1. The greatest labour supply would be provided by the average consumer since she would have the highest marginal utility of consumption. The poor and the rich would supply less. An interesting result, but probably hard to justify.

Combining a flat tax rate and a lump sum transfer effectively results in a progressive tax rate. Note that, for small incomes, the effective tax rate might even become negative i.e. may result in a social benefit payment.

I further restrict the set of possible tax policies to two scenarios. In Section 5.1, redistributive tax policy is implemented by a flat tax on both, capital and labour income. In Section 5.2, I deliberately analyse the question of welfare improvement through capital taxation only.

Both sections restrict the analysis on the utility function u^a which, presumably, is still the most dominant one in the literature of relative consumption. As before, and following Abel (1990), γ is set to one in the relative consumption scenario and to zero in the classical scenario.[32]

5.1 Redistributive total income taxation

In this section, the redistributive tax policy is restricted to the case where $\nu = \nu_k = \nu_n$, i.e. in which the aggregate redistribution amount $D \equiv d$ is financed by a flat tax on labour *and* capital income.[33]

In order to balance the government budget, ν is endogenously given for each d and vice versa. In equilibrium it holds that:

$$\underbrace{\sum_{e\in 0,1} \int_0^\infty df(e,k)dk}_{\equiv D=d} = \underbrace{\sum_{e\in 0,1} \int_0^\infty \nu(rk+ewn)f(e,k)dk}_{\equiv \nu(Y-\delta K)} \tag{31}$$

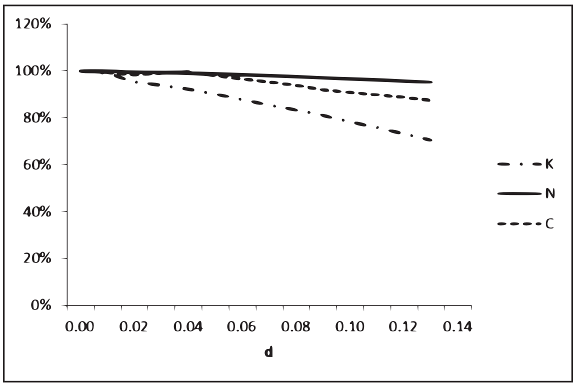
Figure 6 shows how various aggregates, as well as welfare and the GINI coefficient, evolve when a social planner successively increases d, or ν, respectively. Figures 6(a) and 6(b) show the consequences for a classical economy, whereas 6(c) and 6(d) show the same scenario in a relative consumption environment. In the first case, it turns out that implementing a redistributive tax on total income does not raise welfare at all. The tax increase gives rise to the typical disincentive effects with respect to capital accumulation and labour supply. Consequently, the economy suffers from a deadweight loss in capital, output and consumption. In total, aggregate welfare decreases. This implies that the optimal tax policy, among the set of given policies, would be to stick to a non-progressive tax rate with $\nu = d = 0$.

However, if we believe in the effect of relative consumption, the optimal tax strategy turns out to be the opposite. As Figures 6(c) and 6(d) point out, aggregate welfare can be raised by implementing a distributive tax system with $d > 0$.[34] Interestingly, welfare

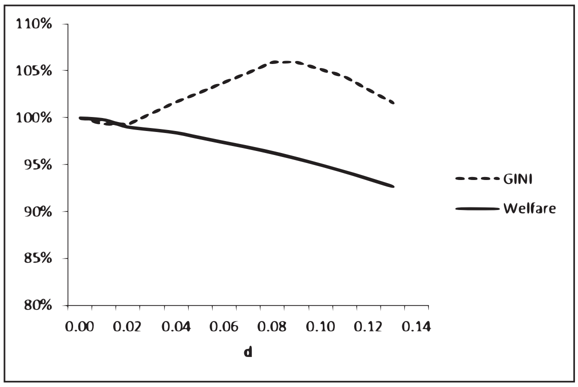
[32] Of course, pursuing the experiment with other utility functions from Section 3.6, or with other values of γ, is straight forward.

[33] Due to the unit mass of households and the fact that every household gets the same lump sum transfer d, the aggregate transfer, D, equals the individual amount.

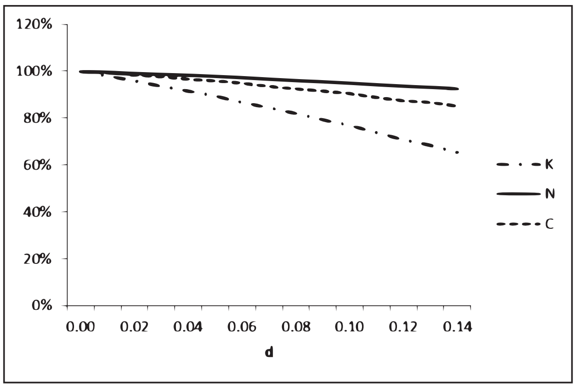
[34] Note that both economies, with and without relative consumption, start from the same situation due to a calibrated adjustment of the weight on labour supply according to (29).



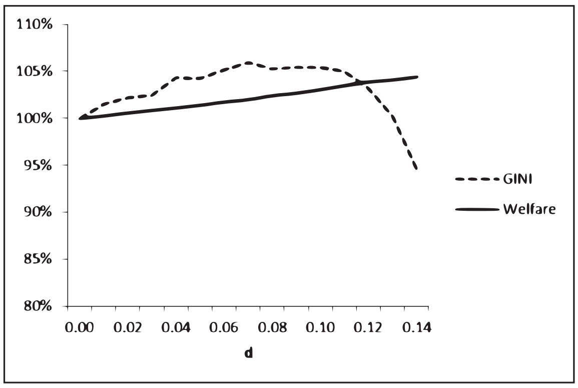
(a) K, N and C for $\gamma = 0$.



(b) $GINI$ and V for $\gamma = 0$.



(c) K, N and C for $\gamma = 1$.



(d) $GINI$ and V for $\gamma = 1$.

Figure 6: Evolution of various aggregates when a redistributive, progressive tax on *total* income is implemented in a classical and in a relative consumption economy.

increases despite the fact that the economy shrinks significantly. Households are willing to accept a lower level of personal income and consumption as long as the *Joneses* - or, the average consumer - reduces his or her level of consumption, too.

An interesting, non-monotonic behaviour can be observed with respect to the wealth distribution in both economies. A rather small progressive tax increases inequality in wealth holdings. In contrast, a more significant redistribution, causes the GINI coefficient to decrease again. Due to the lump sum transfer d, which serves as a social benefit for the poor, all households in general are less anxious about a social descent. Consequently, their incentive to build up precautionary savings decreases, and, as a result, inequality increases. At some point, however, this effect is outperformed by the redistribution effect on the one hand and the disincentive to save due to high taxes, on the other.

The optimal redistributive tax policy in the relative consumption economy is a border solution: namely, the rightmost point in Figure 6(d). If tax progressivity is increased any further, the tax rate ν, which is needed to finance the redistribution amount d, will surpass 100%. In that case, households will stop accumulating any capital at all and the economy will collapse.

5.2 Redistributive capital income taxation

In this section the redistributive tax policy is implemented in the way that the lump sum transfer D is exclusively financed through capital taxation. This analysis is particularly interesting, since the question of taxing capital is usually negated.[35] The exceptions are life cycle and incomplete market models as discussed in Aiyagari (1995) and Conesa et al. (2009). However, the latter emphasize that a positive capital tax is optimal mostly due to the life cycle component, rather than to the existence of incomplete markets. Aiyagari (1995), on the other hand, only uses incomplete markets to show the optimality of a positive capital income tax. However, his model features inelastic labour supply and an additional utility gain from government consumption.

In what follows, it is shown that relative consumption, provides another strong argument for a positive capital tax. In order to pin down the positive effect of capital taxation, the benchmark setting in this subsection has no capital taxes at all, i.e. $\nu_k = 0$ and $\tau_k = 0$. All employment benefits are financed through a tax on labour income, only. During the numerical analysis, d and thereby ν_k will successively be increased, while keeping $\tau_k = 0$, such that taxes on capital income are solely used for redistribution.

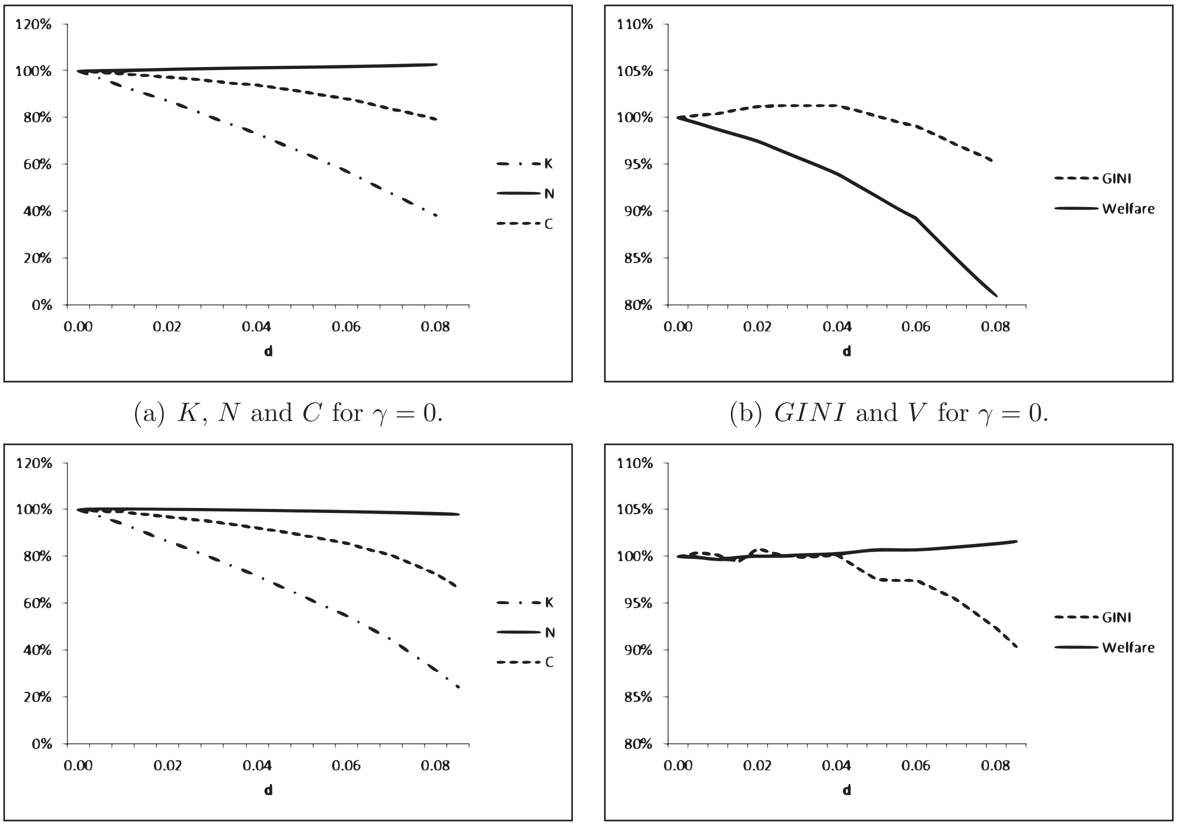
As before, d is endogenously determined by ν_k and vice versa, in order to balance the government budget:

$$\underbrace{\sum_{e\in 0,1}\int_0^\infty df(e,k)dk}_{\equiv D=d} = \underbrace{\sum_{e\in 0,1}\int_0^\infty \nu_k r k f(e,k)dk}_{\equiv \nu_k r K} \tag{32}$$

[35] See for example Judd (1985) and Chamley (1986)

The results of this numerical experiment are shown in Figure 7. As in the previous section, welfare can be increased in the relative consumption economy shown in 7(c) and 7(d). However, the effect is not as pronounced as it had been previously. This is due to the fact, that the lump sum transfer d has to be financed through much smaller income flows which rises the marginal tax rate much faster than in the previous section. High marginal capital taxes lead to a much stronger disincentive to accumulate capital than before. As a consequence, the utility losses due to a shrinking economy are significantly greater than before.

The optimal tax rate in this scenario is again a border solution at the rightmost point. As above, the economy collapses as soon as the marginal tax rate on capital income surpasses 100%.



(a) K, N and C for $\gamma = 0$.

(b) $GINI$ and V for $\gamma = 0$.

(c) K, N and C for $\gamma = 1$.

(d) $GINI$ and V for $\gamma = 1$.

Figure 7: Evolution of various aggregates when a redistributive, progressive tax on *capital* income is implemented in a classical and in a relative consumption economy.

The main argument remains. Optimal taxation is very much dependent on whether relative consumption behaviour is present or not. As outlined in the introduction, there is an overwhelming support for relative consumption preferences. Hence, research into optimal taxation that neglects this phenomenon may produce seriously wrong conclusions.

6 Conclusion and outlook

This paper analysed the general equilibrium effects of incomplete market economies, inhabited by heterogeneous households who value their consumption level relative to that of others. The results have been compared to standard economies in which households value only their own level of consumption. The effects on the shape of the wealth distribution measured by the GINI coefficient can be linked to changes in average relative risk aversion. Utility functions with built-in relative consumption preferences that have been used in the literature cause poor households to be more risk averse and vice versa. Consequently, if average relative risk aversion is above or the same as in standard models, relative consumption leads to more equality in wealth holdings. The poor increase precautionary savings comparatively more than the rich. On the other hand, if relative consumption economies are calibrated such that only rather poor households show typical values of relative risk aversion, an increase in inequality can be shown.

As a second step, I examined welfare optimizing tax policies in a relative consumption economy. It turns out that these economies react differently to tax reforms in comparison to standard economies. Higher proportional taxes on rich households through a progressive tax rate increase welfare, whereas in the standard setting they do not. The utility drawn from a more equal society overcompensates for the usual income and production losses due to the disincentive effects of higher taxes on capital accumulation and labour supply.

Further research should be pursued to establish parameters for existing utility functions that take relative consumption into account. Similar to the research on relative risk aversion, it would be helpful, if the quantitative impact of relative consumption could be assessed. Almost none of the studies that have used the utility functions discussed in this text underpin their assignment of the relative consumption parameter (here γ) on empirical findings.[36] While there is a general consensus that relative consumption matters, the literature has yet to agree on a utility function and corresponding parameter values.

Moreover, it would be interesting to pursue the same analysis that has been carried out in this paper with economic agents that have internal habits, i.e. who care about relative consumption with respect to their own past level of consumption.[37]

In general, if we believe in the presence of relative consumption, and an overwhelming amount of empirical results points in that direction, then a great deal more research could be done. Many established results of micro-founded general equilibrium models might change if 'social' preferences such as relative consumption are employed.

[36] An exception is Alpizar et al. (2005), as mentioned above.

[37] Diaz et al. (2003) have analysed a heterogeneous agent model with internal habits and find that this leads to more equality - hence, the same result as with external habits, shown in this paper. However, an analysis of optimal taxation is still missing.

7 Appendix

7.1 Computing the stationary equilibrium

This list highlights the principal steps of the algorithm used to compute the stationary equilibrium described in Section 3.4.

1. Compute the stationary rate of employment E given the transition probability matrix P. Since the employment process is modelled as a first-order Markov Chain, this step can easily be accomplished by calculating the eigenvectors of P. The mass of employed households E_t and unemployed households $1 - E_t$ evolves according to $[1 - E_{t+1}, E_{t+1}] = [1 - E_t, E_t]P$. The stationary distribution is characterized by $[1 - E, E] = [1 - E, E]P$. Hence, the time invariant probabilities $[1 - E, E]$ are given by the left eigenvector with respect to the eigenvalue 1 of the matrix P.

2. Guess the aggregate capital stock K and the aggregate labour supply N.

3. Use E, K and N to compute the factor prices r and w using their respective marginal products and b by the balanced budget restriction. C can be derived by the market clearing condition (21).

4. Calculate the optimal policy functions of the household $c^*(e, k)$ and $n^*(k)$ by value function iteration.

   (a) Compute the value function for 200 different exponentially distributed grid points k_i between 0 and 14. The maximum capital stock is never binding.

   (b) During each iteration m, for each grid point k_i and each employment state 0 and 1 look for the optimal next period capital stock according to

   $$v_m(e, k_i) = \max_{c,n} u(c, n, C) + \beta \left[p_{e1} v_{m-1}(1, k_i') + p_{e0} v_{m-1}(0, k_i')\right] \tag{33}$$

   with $e \in \{0, 1\}$ and the law of motion for the next period capital stock

   $$k_i' = (1 + (1 - \tau_k)r)k_i + e(1 - \tau_n)wn(c) + (1 - e)b - c \tag{34}$$

   where optimal labour supply is given by the first-order condition:

   $$n(c) = 1 - \frac{\varphi_0}{(1 - \tau_n)wu_c(c)} \tag{35}$$

   due to the optimality condition (24). The maximum is found by using the golden section search algorithm[38] which is much faster and more precise than grid search. Linear interpolation is used for value function values which are

[38] See, for example, Heer & Maussner (2004), Chapter 11.6.1

not on the grid. The resulting policy functions are smoothed using a moving average procedure. The latter step is not necessary but allows for a smaller grid of k which improves computational speed.

5. Given the optimal policy functions and a randomly drawn Markov Chain based on P, simulate 10000 households until a stationary capital and labour supply distribution is reached. Usually, this takes between 2000 to 6000 steps. The optimal policy functions are interpolated using cubic splines for points off the grid.

6. From the steady-state capital and labour distribution, compute the aggregates K and N.

7. Stop the procedure, if the differences between the simulated values of capital and labour differ at most by half a percent from the guessed values in step 2.

8. If not, determine new values for K and N using a weighted average of the previous guess and the simulation outcome according to

$$K_{new} = \phi_K K_{old} + (1 - \phi_K) K_{simulation}$$

$$N_{new} = \phi_N N_{old} + (1 - \phi_N) N_{simulation}$$

and go back to step 3. $\phi_K = 0.925$ and $\phi_N = 0.875$. The algorithm detects oscillations, in which case it automatically increases these weights. On the other hand, if no oscillations occur, the weights are successively decreased, in order to speed up convergence. Figure 8 shows how this procedure typically converges.

7.2 Calibration of η and γ_0

In order to determine the parameter η that results in an average relative risk aversion of 2, the algorithm described in 7.1 is embedded in another loop iterating over different values of η until the average relative risk aversion computed from the simulated stationary distribution, matches the required value.

The same procedure is used to determine γ_0, in order to calibrate for a specific level of aggregate labour supply.

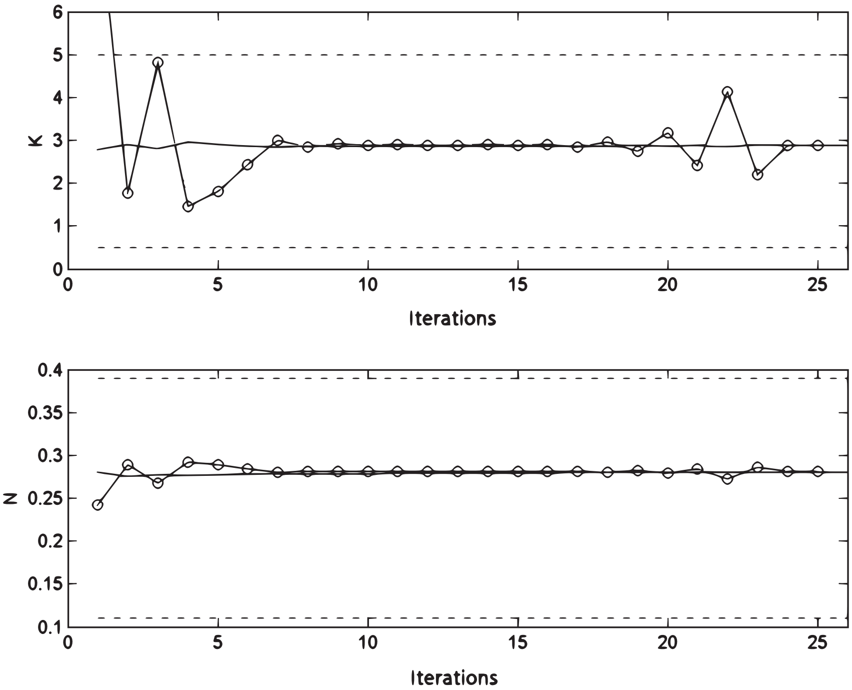


Figure 8: Finding the market clearing values for K and N. The rather flat line in the background displays the guessed values, whereas the circled line shows the simulation values.

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