Exploiting High Frequency Data for Volatility Forecasting and Portfolio Selection

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The President:

Prof. Dr. Thomas Bieger
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3.1 Time Series of the Volatility Asymmetry ..................... 94
An instant may matter for the course of an entire life. It is with this idea that the present research had its inception. High frequency financial data are becoming increasingly available and this has triggered research in financial econometrics where information at high frequency can be exploited for different purposes. The most prominent example of this is the estimation of financial volatility.

Based on the notion of increasing sampling frequency Andersen et al. (2001b) and Barndorff-Nielsen and Shephard (2002b) formalized the realized variance. As volatility is unobservable, proxies are needed in order to model its dynamics. The conventional approach, before the advent of the realized measure, is to use model based estimates of the conditional variance, such as GARCH-type models, that consider squared returns as a proxy for the conditional variance. However, this measure is a very noisy proxy for the variance that is overcome by the realized variance, which uses higher frequency return information. Under the assumption that the return generating process is a continuous semimartingale, the realized variance is a consistent estimator for the integrated variance. Denoting with \( s_t \) the logarithmic asset price at time \( t \), for \( t \in [0, T] \), the return process is

\[
    ds_t = \mu_t dt + \sigma_t dW_t, \tag{1}
\]

where \( \mu_t dt \) and \( \sigma_t dW_t \) represent the time varying mean and stochastic volatility, respectively. Given discretely sampled data, the realized variance for period \( t \) (for instance representing one day) is \( RV_t = \sum_{j=1}^{m} r_{t,j}^2 \), where \( m \) is
the number of high frequency return observation in the period \( t \), and it estimates the integrated variance:

\[
RV_t \xrightarrow{p} \int_{t-1}^{t} \sigma_t^2 d\tau \quad t = 1, \ldots, T.
\]  

The estimation of volatility based on high frequency data is a cornerstone for research in financial econometrics as many applications in finance, ranging from portfolio selection to risk management require an estimate and a forecast of it. Therefore, given that the realized measure is a precise estimate for volatility, a forecasting model for it is of practical interest. A first contribution of this thesis is the forecast comparison of univariate models for the realized volatility in light of several stylized facts of financial time series. Of practical matter is to assess the economic value added of forecasting models that use high frequency information. For this purpose, a second contribution of this thesis is to introduce and to compare multivariate volatility models that takes return information at low frequency, high frequency or a mix of the two frequencies, based on their abilities to generate an economic value added. Finally, high frequency data may be used not only for the forecasting of volatility, but it may contain precious information to price assets and to select portfolios of assets. The third contribution of this thesis is the finding of a pricing factor constructed from high frequency data that has predictive power for expected portfolio returns. The research, chapter by chapter is summarized below.

Chapter 1, which is coauthored with Professor Francesco Audrino, provides empirical evidence on univariate realized volatility forecasting in relation to asymmetries present in the dynamics of both return and volatility processes. It examines leverage and volatility feedback effects among continuous and jump components of the S&P500 price and volatility dynamics, using recently developed methodologies to detect jumps and to disentangle their size from the continuous return and the continuous volatility. The research finds that jumps in return can improve forecasts of volatility, while jumps in volatility improve volatility forecasts to a lesser extent.
Moreover, disentangling jump and continuous variations into signed semi-variances further improves the out-of-sample performance of volatility forecasting models, with negative jump semivariance being highly more informative than positive jump semivariance. A simple autoregressive model is proposed and this is able to capture many empirical stylized facts while still remaining parsimonious in terms of number of parameters to be estimated.

Chapter 2 investigates the out-of-sample performance and the economic value of multivariate forecasting models for volatility of exchange rate returns. It finds that, when the realized covariance matrix approximates the true latent covariance, a model that uses high frequency information for the correlation is more appropriate compared to alternative models that uses only low-frequency data. However multivariate FX returns standardized by the predicted high frequency multivariate volatility are not normally distributed. Using high frequency information for the correlation, the multivariate model tends to produce forecasts of tail risk which are lower than the realized tail risk, under the normality assumption.

Chapter 3 finds that a factor, constructed from high frequency market price data and representing the volatility asymmetry, is able to explain a significant proportion of the time and cross sectional variation of average returns. Small, growth and high beta portfolios are particularly subject to the asymmetry risk. A multi-factor pricing model with the high frequency volatility asymmetry is proposed and it performs well in pricing tests.

The research literature related to this thesis consists of well defined sub-fields in financial econometrics and applied finance. Chapter 1 and Chapter 2 are most closely related to the literature on volatility modeling and the assessment of the model forecast accuracy. The obvious delimiting difference between the two chapters is that Chapter 1 focuses on univariate high frequency volatility models, while Chapter 2 focuses on multivariate GARCH-type models. Chapter 3 is related to the literature on empirical asset pricing and it is intimately related to Chapter 1 as it reconciles a stylized fact, the so called asymmetric volatility, with the pricing of return portfolios. All the
three chapters are centered on the use of high frequency intra-day data, the usefulness of which is the central message of the thesis as a whole. Finance applications of high frequency data are presented in Chapter 2, for portfolio selection and risk management, and in Chapter 3, for asset pricing.

The thesis is related to the mainstream literature of time series models for volatility. Let alone, it is a vast field consisting of hundreds of published papers over the past three decades, most of them focusing on GARCH-type models. Poon and Granger (2003) offer an excellent overview of volatility models in the univariate framework, while Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009) survey multivariate GARCH models. A more recent type of models for volatility is so-called high frequency models based on the realized measure. The attractiveness of those models relies on the premise that with an accurate measure for volatility, such as the realized volatility, one needs only a model for its expectation and current volatility may be treated as observed. This substantially reduces the model complexity so that simple autoregressive models may result appropriate in this framework. Popular autoregressive models for the realized volatility are the ones capable of capturing the long memory property of the time series. Those are the ARFIMA model, which is first used to model the realized volatility by Andersen et al. (2003), and the HAR – RV model of Corsi (2009). Based upon the HAR – RV model, Chapter 1 extends this literature and considers several other stylized facts in the dynamics of the return and volatility processes that need to be accounted for in forecasting volatility.

A specialized literature deals with model forecast comparisons. The present research contributes empirically to this literature. In line with Hansen and Lunde (2006a), Patton (2011), and Laurent et al. (2009), the realized measure is used as a proxy for the true ex-post volatility. The methodological development of the previous literature, in terms of the choice of the loss functions and testing procedures to yield ranking of volatility models is widely used in the my research. More specifically, the model confidence set test of Hansen et al. (2011) is the primary methodology used in order to yield the
model forecast comparison results presented in Chapter 1 and Chapter 2.

Asset pricing, portfolio selection, and risk management are areas of application for high frequency volatility estimation and forecast that are presented in this thesis. First, based upon the CAPM and the Fama and French (1992) model, the thesis moves in the direction of integrating a high frequency measure into the pricing model. This is motivated by the presence of patterns in the dynamics of the return process that can be more appropriately captured with high frequency information. Second, the thesis presents a volatility timing strategy to select portfolios with forecasts of the multivariate volatility based on high frequency data. Finally, a backtesting risk assessment is executed in order to evaluate the model performance in a risk management perspective. Those applications illustrate practical contexts for which the whole research can be directly applied. Another potential application of high frequency data is in the context of risk measurement. For instance, downside risk and jump inference as introduced in Chapter 1 are made possible only with high frequency data. Those measures of risk may yield a characterization of the market risk which is more comprehensive.
1.1 Introduction

Volatility forecasts are crucial for many investment decisions. They are relevant for option pricing, asset allocation, and risk management. Accurate estimates of volatility are essential prerequisites for good forecasts. In light of this, the development of high frequency estimators, based on the notion of increasing sampling frequency, has put the research on this field a step forward. In contrast with model-based estimates, the realized volatility, advocated by Andersen et al. (2001a) and Barndorff-Nielsen and Shephard (2002a), among others, consistently estimates the integrated volatility of the return process under the assumption that it follows a continuous semimartingale. Thus, realized measures represent the foundation of any forecast of volatility. Supporting this, Hansen and Lunde (2006a) suggest that model comparisons be based on the use of the realized volatility as a proxy for the latent volatility, and the results in Andersen et al. (2003) indicate that simple autoregressive forecasting models based on the realized volatility outperform GARCH related models in an out-of-sample perspective.
Many stylized facts are known for return and volatility dynamics. Those include volatility persistence, volatility leverage and feedback effects, and jumps that induce skewness and leptokurtosis on the return and volatility distributions. The existence of these effects poses challenges for volatility forecasts by requiring models that can account for those empirical features. In this regard, forecasting models based on non-parametric estimates of volatility are particularly suited as jump information can be extracted directly from the data and volatility persistence, leverage and feedback effects can be modeled parsimoniously. Capturing volatility persistence, the popular heterogeneous autoregressive realized volatility model (HAR – RV) of Corsi (2009), which is based on an approximation of long run volatility components, performs surprisingly well in out-of-sample forecasts.

This chapter sheds light on those stylized facts that are useful for volatility forecasting. We consider volatility leverage effects of continuous and jump components of the return/volatility dynamics as key ingredients of the forecasting models we propose. Particularly relevant for the analysis of asymmetries between jump components is the methodology to detect intraday return jumps proposed by Lee and Mykland (2008). Moreover, in the spirit of Corsi (2009), we model the long memory of volatility with HAR components.

A discrepancy exists in the interpretation of leverage and feedback effects. In option pricing and continuous time models, the two effects materialize in the same way even if the causes differ. They are both commonly interpreted as a negative contemporaneous correlation between changes in log-price and volatility (see Mykland and Zhang, 2009; Bandi and Renò, 2010). In the discrete time literature, a fundamental distinction exists between the two effects and this is inherent to the timing of the correlation. The leverage effect arises when there is negative correlation between volatility and lagged returns that is originally motivated by the effect of the financial leverage. A negative shock in the stock price leads the financial leverage to increase and, in turn, volatility as well, as it is considered an increasing
function of the leverage (see Christie, 1982; Schwert, 1989). In contrast, the volatility feedback effect materializes in the opposite causal direction. The common motivation is that an increase in volatility is associated with an expectation of higher future volatility and, therefore, market participants discount this information, resulting in an immediate drop in stock prices (see French et al., 1987; Campbell and Hentschel, 1992; Bekaert and Wu, 2000; Wu, 2001; Bae et al., 2007, among others). In discrete time models for volatility forecasting, this effect is of minor importance as, ultimately, it has to do with returns. However, persistency in volatility is a channel through which this effect manifests itself, and therefore the discrete time model proposed accounts not only for the leverage effect but also for the volatility feedback effect.

Evidence of the leverage effect with high frequency data has been advanced by Bollerslev et al. (2006) who analyzed sign asymmetries of high frequency returns and their impact on future volatility. This chapter presents new evidence of leverage and volatility feedback effects by using the S&P500 index. To the prior literature, it adds consideration of a separate leverage effect for the jump and continuous components of the return and volatility processes. These are analyzed by means of cross-correlation between realized variation and return, with both realized variation and return being disentangled into continuous and jump components. We find that the leverage effect originates from both continuous and jump components. However, the dynamics differ as continuous component correlations tend to persist for a prolonged period, while correlation of jump components is short-lived.

The main contribution of this chapter is the finding of a superior forecasting performance when the jumps in returns are isolated from the corresponding continuous component. While the previous papers separated jump variation and continuous variation, as proposed by Andersen et al. (2007a), here we disentangle the jump size from the continuous return. With this approach, any effect of return jumps on future volatility has a clear interpretation as a jump leverage effect. Consistent with Corsi and Renò
(2010), the model proposed sheds light on the leverage effect also by taking lagged negative returns. Further, in agreement with Patton and Sheppard (2011), we find that the best forecasting model includes “downside risks,” which are volatilities generated by negative intra-day returns. The realized variation is in fact also disentangled into continuous signed semivariations and jump signed semivariations, as a way to capture separate dynamics of negative intra-day returns with respect to positive intra-day returns. The leverage effect in forecasting realized volatility has been considered by both Corsi and Renò (2010) and Patton and Sheppard (2011) in a similar framework. These papers are the most closely related to our study. However, the methodology to assess it differs as we disentangle jumps size from both return and volatility, and we also separate both jump variation and continuous variation into semivariances.

The remainder of the chapter is organized as follows. Section 1.2 presents realized estimators used in the analysis, the continuous time framework on which the estimates are based, and the methodology to disentangle continuous and jump components from both return and volatility processes. Section 1.3 presents the data used in the empirical analysis and the candidate forecasting models. Section 1.4 contains evidences for the leverage and volatility feedback effects and presents estimation results and the out-of-sample forecasting performance evaluation. Finally, Section 1.5 concludes.

1.2 Theoretical Framework

The underlying framework for the empirical analysis is based on a double jump-diffusion data generating process. This stochastic process features a continuous sample path component and occasional jumps in both return and volatility dynamics. The framework was first laid down by Duffie et al. (2000). The empirical analysis of this class of models can be found in Broadie et al. (2007), Chernov et al. (2003), Eraker et al. (2003), Eraker (2004), and Todorov and Tauchen (2011). Generally, the previous results support jumps
in volatility as well as jumps in returns for speculative prices. Let $s_t$ denote the logarithmic asset price at time $t$, for $t \in [0, T]$. In stochastic differential equation form, the price and volatility processes are

$$
\begin{align*}
\text{d}s_t &= m_t \text{d}t + \sigma_t \text{d}W^s_t + \kappa_t^s \text{d}J^s_t, \\
\text{d}\sigma^2_t &= \vartheta \left( \theta - \sigma^2_t \right) \text{d}t + \sigma^v_t \text{d}W^v_t + \kappa^v_t \text{d}J^v_t,
\end{align*}
$$

where $\{W^s_t, W^v_t\}$ is a bivariate standard Brownian motion, $\{dJ^s_t, dJ^v_t\}$ is a bivariate count process and $\{\kappa^s_t, \kappa^v_t\}$ represents the size of the jumps in return and in volatility if a count occurs at time $t$, with $\kappa^v_t$ restricted to be non-negative\(^1\). The mean process of the squared volatility equation (1.2) is characterized by a long run level parameter $\theta$ and a mean-reverting parameter $\vartheta$. Moreover the instantaneous volatility of the squared volatility, $\sigma^v_t$, is allowed to be time-varying.

This framework allows the generation of the contemporaneous leverage or volatility feedback effects through the correlation between the continuous components as well as through the correlation between the jump components that may be both in time and in size of the jumps. However, with the discrete time analysis we propose, leverage and volatility feedback effects are interpreted as lag correlation and therefore the assumption of stochastic dependence is not needed.

Following the theory of quadratic variation, the volatility of the price process is estimated with the realized volatility from high frequency data. Below, we present the estimators for volatility and jump components used in the analysis.

\(^1\)It is common in the option pricing literature to assume a compound Poisson process, where jump arrivals have a Poisson distribution with intensity $\{\lambda^s, \lambda^v\}$ and jump sizes in return have a normal distribution. In the present context these assumptions are not needed.
1.2.1 Return Volatility and Jumps

Let $r_{t,i}$ be the discretely sampled $i$-th intra-day return for day $t$. In the presence of jumps in return, the realized variation, $RV_t = \sum_{i=1}^{m} r_{t,i}^2$, introduced by Andersen et al. (2001a,b), captures both continuous and jump components of the quadratic variation:

$$RV_t \overset{p}{\rightarrow} \int_{t-1}^{t} \sigma_\tau^2 d\tau + \sum_{t-1<\tau\leq t} (\kappa_\tau^s)^2 \quad t = 1, \ldots, T. \quad (1.3)$$

The bipower variation, introduced by Barndorff-Nielsen and Shephard (2004b), is instead a consistent estimator for the continuous component only:

$$BV_t = \mu^{-2} \sum_{i=2}^{m} |r_{t,i}| |r_{t,i-1}| \overset{m\rightarrow\infty}{\rightarrow} \int_{t-1}^{t} \sigma_\tau^2 d\tau \quad (1.4)$$

where $\mu = 2^{1/2} \frac{\Gamma(1)}{\Gamma(1/2)} = \sqrt{2/\pi}$, with $\Gamma(\cdot)$ denoting the gamma function.

Andersen et al. (2007b) and Lee and Mykland (2008) propose detecting intra-day jumps using the ratio between intra-day returns and estimated spot volatility. We follow the methodology laid down by Lee and Mykland (2008) and, in particular, we test the presence of intra-day jumps with the statistics

$$L_{t,i} = \frac{r_{t,i}}{\left(\frac{1}{K-1} \sum_{k=1}^{K-1} |r_{t,i-k}| |r_{t,i-k-1}|\right)^{1/2}}, \quad (1.5)$$

where the expression contains in the denominator the estimates of the spot volatility as the average bipower variation over a period with $K$ observations. Lee and Mykland (2008) suggest using $K = 78, 110, 156, 270$, respectively, with return sampled at frequencies of 60, 30, 15, and 5 minutes. Under the null of no intra-day jump, the test statistics $L_{t,i}$ follow a normal distribution (with variance $\mu$).

In order to select the rejection region for the test statistics $L_{t,i}$, Lee and Mykland (2008) propose to look at the asymptotic distribution of maxima
of the test statistics. As the sampling frequency tends to zero, under the null of no jumps between time \((t, i - 1)\) and \((t, i)\), the absolute value of \(\mathcal{L}_{t,i}\) converges to a Gumbel distribution:

\[
\max_{(t,i)} |\mathcal{L}_{t,i}| - C_n \xrightarrow{d} \zeta, \tag{1.6}
\]

where \(\zeta\) has a standard Gumbel distribution, \(C_n = \frac{(2 \log n)^{1/2}}{\mu} - \frac{\log \pi + \log(\log n)}{2\mu(2 \log n)^{1/2}}\), \(S_n = \frac{1}{\mu(2 \log n)^{1/2}}\), and \(n\) is the number of observations for each period \(t\). We reject the null of no jump at time \(t, i\) if

\[
\frac{|\mathcal{L}_{t,i}| - C_n}{S_n} > \beta^*, \tag{1.7}
\]

such that \(\exp\left(-e^{-\beta^*}\right) = 1 - \alpha\), i.e. \(\beta^* = -\log\left(-\log\left(1 - \alpha\right)\right)\), with \(\alpha\) being the significance level\(^2\).

The test is able to detect the jump arrival time \(i_j\) for each day \(t\), where \(j\) denotes the presence of a jump. Moreover the jump size is computed as

\[
\kappa_{t,i_j} = \left(r_{t,i_j}\right) \mathbb{1}\left\{\frac{|\mathcal{L}_{t,i}| - C_n}{S_n} > \beta^*\right\}. \tag{1.8}
\]

Consequently, the jump size for the day \(t\) is

\[
jRet_t = \sum_{i_j = i_1, \ldots, i_{J_t}} \kappa_{t,i_j}, \quad t = 1, \ldots, T, \tag{1.9}
\]

where \(J_t\) is the total number of significant jumps for day \(t\), and the jump-adjusted daily return is

\[
cRet_t = r_t - jRet_t, \quad t = 1, \ldots, T. \tag{1.10}
\]

\(^2\)The extant literature has proposed other compelling tests for the presence of jumps. Alternative tests are mainly based on the difference between \(RV_t\) and \(BV_t\) and follow the asymptotic distribution theory of Barndorff-Nielsen and Shephard (2006). In a robustness check, we obtain the same empirical results regarding forecasting performance of our models with both tests for significant jumps of Barndorff-Nielsen and Shephard (2006) and Corsi et al. (2010), combined with the recursive methodology of Andersen et al. (2010) to identify the size of each intra-day jump.
Chapter 1. Volatility Forecasting: 
Downside Risk, Jumps and Leverage Effect

With this methodology to identify intra-day jumps, it is possible to directly estimate the jump variation, i.e. the quadratic variation of return jumps. We follow Andersen et al. (2010) and estimate the quadratic variation due to the continuous and the jump components respectively as

\[ CV_t = RV_t - JV_t, \quad t = 1, \ldots, T, \]  
\[ JV_t = \sum_{j=1}^{J_t} JV_{t,j}, \quad t = 1, \ldots, T, \]

where

\[ JV_{t,j} = \mathbb{1}_{\{ \kappa_{t,j} \neq 0 \}} \left( \kappa_{t,j}^2 - \frac{1}{m - J_t} \sum_{k \in \{1, \ldots, m\} \setminus \{i_j, \ldots, i_J\}} r_{t,k}^2 \right), \]

is the contribution to the quadratic variation of each intra-day jump \( \kappa_{t,j} \), with \( j = 1, \ldots, J_t \) and \( t = 1, \ldots, T \). We find that the daily jump variation identified with this methodology is highly correlated with more traditional methods to identify jump variation that primarily take the positive differences between \( RV_t \) and \( BV_t \).

1.2.2 Downside Continuous and Jump Variation

To capture the sign asymmetry of the volatility process, the continuous variation and jump variation are decomposed using signed intra-day returns. Barndorff-Nielsen et al. (2010) introduced a new estimator that captures the quadratic variation due to signed returns, termed realized semivariance. In a similar way, the continuous variation and jumps variation can both be decomposed into signed semivariations by using the test of Lee and Mykland (2008). This represents the main advantage of the test, as it is able to identify the sign and the timing of intra-day jumps. If jump variation is instead identified by means of significant \( (RV_t - BV_t) \), as normally done in the previous literature, no-information is available about the sign or timing of intra-day jumps and therefore we are not able to disentangle jump or continuous semivariations.
1.2. Theoretical Framework

The realized semivariances of Barndorff-Nielsen et al. (2010) are defined as

\[
RS_t^+ = \sum_{i=1}^{m} r_{t,i}^2 \mathbb{1}_{\{r_{t,i} > 0\}}, \quad (1.14)
\]

\[
RS_t^- = \sum_{i=1}^{m} r_{t,i}^2 \mathbb{1}_{\{r_{t,i} < 0\}}. \quad (1.15)
\]

Similarly, the jump semivariation is generated by signed intra-day jumps and it is defined as follows:

\[
JSV_t^+ = \sum_{j=1}^{J_t} JSV_{t,j}^+, \quad (1.16)
\]

\[
JSV_t^- = \sum_{j=1}^{J_t} JSV_{t,j}^-, \quad (1.17)
\]

where

\[
JSV_{t,j}^+ = \mathbb{1}_{\{\kappa_{t,j} > 0\}} \left( \kappa_{t,j}^2 - \frac{1}{m - J_t} \sum_{k \in \{1,...,m\} \setminus \{i_j,...,i_{jt}\}} r_{t,k}^2 \right), \quad (1.18)
\]

\[
JSV_{t,j}^- = \mathbb{1}_{\{\kappa_{t,j} < 0\}} \left( \kappa_{t,j}^2 - \frac{1}{m - J_t} \sum_{k \in \{1,...,m\} \setminus \{i_j,...,i_{jt}\}} r_{t,k}^2 \right), \quad (1.19)
\]

for \( j = 1, \ldots, J_t \) and \( t = 1, \ldots, T \). Consequently, the continuous semivariation is given by

\[
CSV_t^+ = RS_t^+ - JSV_t^+, \quad (1.20)
\]

\[
CSV_t^- = RS_t^- - JSV_t^- . \quad (1.21)
\]

The decomposition in semivariances is complete as \( RV_t = RS_t^+ + RS_t^- \), \( JV_t = JSV_t^+ + JSV_t^- \) and \( CV_t = CSV_t^+ + CSV_t^- \). To study the volatility feedback effect, a particular focus is on the downside continuous variation and jump variation captured by \( CSV^- \) and \( JSV^- \), respectively, as Patton and
Sheppard (2011) show that negative semivariances are more informative than positive semivariances for forecasting future volatility. These quantities are used in the empirical analysis of the following sections.

1.2.3 Volatility Jumps

The estimation of volatility jumps is a prominent topic of research (see Todorov and Tauchen, 2011). Parametric methods can be employed to estimate the jump intensity parameter and the average jump size for the process (1.2) given a distributional assumption on the jump process. However, we consider an alternative methodology to estimate the size of (continuous) volatility jumps without imposing assumptions on its distribution. As is standard practice in the realized volatility literature, the volatility estimate (continuous variation, equation 1.11) is taken as observed.

We consider an auxiliary AR(1) model on the level of continuous variation with GARCH(1, 1) dynamics. This choice is motivated empirically and theoretically. The AR – GARCH on the changes in continuous volatility is a natural discrete approximation of the volatility process in (1.2) without the volatility jump component, and, moreover, it fits very well to the continuous variation series by excluding extreme values. It is specified as

\[
(\Delta CV)_t = c + \phi \cdot CV_{t-1} + u_t, \tag{1.22}
\]

\[
u_t = \sigma^v_t e_t, \quad e_t \sim N(0, 1), \tag{1.23}
\]

\[
(\sigma^v_t)^2 = \omega + \alpha \cdot u^2_{t-1} + \beta \cdot (\sigma^v_{t-1})^2, \tag{1.24}
\]

with stationarity constraint $|\phi| < 1$, $(\alpha + \beta) < 1$, and non-negativity constraints on $(\sigma^v_t)^2$: $\omega > 0, \{\alpha, \beta\} \geq 0$. The jump in volatility is estimated as

\[
VolJ_t = \mathbb{1}_{\{e_t > \Phi_{1-\alpha}\}} \cdot e_t, \tag{1.25}
\]

where $\Phi_{1-\alpha}$ denotes the $1 - \alpha$ quantile of the standard normal distribution.
Intuitively, in the absence of volatility jumps, the auxiliary model approximates the true volatility dynamics. However, in the presence of large jumps, the model is not able to fit the data and large residuals from the model fit represent contributions of volatility jumps. The jump is in the continuous variation; this combination of terms sounds somehow antithetical. The continuous variation captures the volatility without jumps in returns. However, it may well be that it presents jumps itself, whether or not there are jumps in returns. Jumps in continuous variation are therefore labeled volatility jumps. The continuous part of the continuous variation is labeled adjusted continuous variation and it is given by

\[
adjCV_t = CV_t - VolJ_t.
\]  \hspace{1cm} (1.26)

To maintain positivity of the adjusted continuous variation, a restriction is made on the volatility jumps. It may happen that the estimated volatility jumps, \(VolJ_t\), are higher than the total continuous variation \(CV_t\). In such cases, the volatility jumps are set equal to the continuous variation such that \(adjCV_t\) equals zero. The interpretation of this is that the volatility on certain specific days is entirely driven by jumps.

### 1.3 Data and Forecasting Models

#### 1.3.1 Data and Summary Statistics

The data used in the analysis consists of tick-by-tick prices for the Standard & Poor (S&P500) Future. The sample period covers almost the entire history of this security: from April 28, 1982 to August 6, 2010. The S&P500 Future is traded “open outcry” on regular market opening hours of the Chicago Mercantile Exchange (from 8:30 AM to 15:15) and it has also been used by Bollerslev et al. (2009) and Corsi (2009), among others, as an index for the composite market.
The data cleaning procedure follows Hansen and Lunde (2006b). In particular, as trades before 9:00 AM for the first several years from the introduction of this security are often not very active, all the trades outside 9:00 AM and 15:15 are discarded. Transactions with zero volume are also discarded. Moreover, days with less than five consecutive trading hours are also discarded. Prices are then sampled every five minutes, starting from 9:00 AM, in order to smooth the impact of market microstructure noise. Furthermore, whenever there is no observed price attached to a specific time stamp, the “previous tick” method is used to replace the missing price.

Figure 1.1 represents the time series of returns, log realized variation and jumps in return and volatility. The highest levels of realized volatility were reached on “black Monday”, October 19, 1987, and the following day October 20, where returns were respectively about -26% and -9.6%. The test (3.20) detects jumps during both days. However for October 19, 1987, the intra-day price changes were less drastic than on the following day, resulting in a lower jump variation. In October 20, 1987, jump variation was at the highest level. During that day negative jumps were combined with positive jumps, while on October 19, sequences of negative intra-day returns dominated. The jump tests are carried out through the analysis with $\alpha = 0.01$ significance level.

Jumps in volatility are also identified for both October 19 and 20, 1987, with the volatility jump for October 19 being at the highest level. Accordingly, the market crash of October 19, 1987 is explained with jumps in returns and even more with a jump in volatility. This underscores the importance of allowing for volatility jumps in order to fully capture the dynamics of extreme events. Return jumps alone, identified with the testing methodology, are not able to fully account for extreme market events. Summary statistics for the intensity of jumps in return and in volatility are reported in Table 1.1.

On average, there is less than one return jump per day. With significance level $\alpha = 0.01$ and five minutes sampling, the number of days with at least
one intra-day jump represents 19.29% of the sample period. The intra-day jump intensity ranges from 0 to 10 jumps (maximum corresponding to October 19, 1987). Conditional on the presence of jumps there are on average 1.43 intra-day return jumps each day. Regarding jumps in volatility, we find

![Figure 1.1: Time Series of Return, Volatility, and Jumps](image)

NOTE: The figure plots time series of daily log return, log realized variation, jump in returns and log jump in continuous variation. The underlying security is the S&P 500 index Future and the time period ranges from April 28, 1982 to August 6, 2010.
an intensity of about 3.65% of the trading days. The analysis for volatility jumps is also carried out at significance level $\alpha = 0.01$ in equation (1.25). As with jumps in return, other significance levels are experimented. The chosen cutoff level $\alpha = 0.01$ is empirically motivated by the “quantile-quantile” Plot showed in Figure 1.2.

**Figure 1.2: Volatility Jumps as Extreme GARCH Residuals**

![QQ Plot of Residuals VS Standard Normal](image)

NOTE: The figure represents the quantiles of the standardized residuals from the $AR – GARCH$ fitting on the continuous variation against the quantiles of the standard normal distribution. The red dots represent the quantiles of the standard normal distribution against itself. Extreme values that deviates from the normal assumption are volatility jumps.

The empirical distribution with this cutoff level best approximates that of a standard normal distribution. Without a threshold, the residuals from the auxiliary $GARCH$ model fitting deviate substantially from the assumed standard normal distribution. Only large departures from the standard normal distribution are categorized as volatility jumps. The contribution of return jumps to total return levels, the relative contribution of jump variation to the total variation and the contribution of volatility jumps to the continuous volatility are reported in Figure 1.3.
The contribution of negative jumps to negative returns appears to be higher, on average, than the contribution of positive jumps to the corresponding signed return. This jump sign asymmetry may account for the negative skewness observed in financial markets. Based on rolling averages of 3 months and 1 year windows, the contribution of the negative jumps to corresponding signed returns ranges from approximately 0.1% to 4.5%, on 3 months basis, and from 1% to 3.5%, on a yearly basis, while that of positive jumps ranges from 1% to 2.5%, on a yearly basis. Jump variation accounts for approximately 3% to 7% on a yearly basis. This statistic is in line with the ones reported by the previous literature despite the use of a different methodology to test for the presence of jumps. Corsi and Renò (2010), for example, report an average percentage over almost 28 years of about 6% for the S&P500 index, and Andersen et al. (2010) report an average percentage over five years that ranges from 2.1% to 5.8% for different stocks. Regarding volatility jumps, they produce a minor contribution to the continuous variation as they are more sporadic than return jumps. They account for about 0% to 8% of the continuous variation on a yearly basis, and for the entire sample period from 1982 to 2010 they represent 3.4% of continuous variation.

1.3.2 Forecasting with Jumps, Leverage effect, and Volatility Persistency

Given the measures of volatility and jump magnitude introduced previously, it is of interest to study their additional forecasting power for future volatility. The models proposed are based on the Heterogeneous Autoregressive (HAR) framework of Corsi (2009) and extend it by considering semivariances, jumps in return and in volatility, and the leverage effect due to continuous return and return jumps.

The realized volatility features long memory. The HAR model, although it does not belong formally to the class of long-memory models, represents a
parsimonious approximation, which is able to closely mimic such a stylized fact. In more detail, the benchmark model is

$$\log (RV)_{t,t+h} = \alpha + \phi^d \log (RV)_t + \phi^w \log (RV)_{t-5,t} + \phi^m \log (RV)_{t-22,t} + \epsilon_t,$$  \hspace{1cm} (1.27)

where $\log (RV)_{t,t+h}$ is the average log realized variation between time $t$ and $t + h$. The variables $\log (RV)_{t-5,t} = \frac{1}{5} \sum_{i=t-4}^{t} \log (RV_i)$ and $\log (RV)_{t-22,t} = \frac{1}{22} \sum_{i=t-21}^{t} \log (RV_i)$ capture long memory features of the volatility process. The coefficients of the model may be interpreted as the reaction of heterogeneous agents who forecast with different time horizons: daily, weekly and monthly. The representation of the HAR forecasting model based on realized variation instead of log realized variation,

$$RV_{t,t+h} = \alpha + \phi^d RV_t + \phi^w RV_{(t-5,t]} + \phi^m RV_{(t-22,t]} + \epsilon_t,$$ \hspace{1cm} (1.28)

is also considered. However, the last produces inferior forecasts and therefore results are summarized and discussed for the log-log model.

Several model extensions based on the HAR framework have been proposed in the recent literature: see, among others, Andersen et al. (2007a), Corsi and Renò (2010), and Patton and Sheppard (2011). Andersen et al. (2007a) included jump variation in forecasting volatility dynamics. Corsi and Renò (2010) studied the impact of the leverage effect on volatility by including past signed returns. Finally, Patton and Sheppard (2011) and Chen and Ghysels (2010) considered the use of realized semivariances to forecast volatility. None of the previous studies, however, used the return jumps themselves and jumps in volatility to generalize the HAR model.

Jumps in return arguably have an impact on future volatility. Contrary to the effect of jump variations, this is interpreted as a leverage effect, which can manifest itself differently whether it comes about through continuous returns or jumps. In fact, although both components together generate the
final leverage effect, their dynamics differ, as jumps have a very short impact on future volatility while continuous returns tend to have a persistent impact on future volatility.

Disentangling jump variation and continuous variation systematically improves volatility forecasts, and disentangling downside semivariation and upside semivariation also improves forecasts. We combine the ideas of Andersen et al. (2007a) and Patton and Sheppard (2011) by considering a complete decomposition into signed continuous and jump variation and this leads to even better forecasts. Negative jump semivariation has in fact a completely different impact on future volatility than positive jump semivariation. Lastly, jumps in volatility may also improve future volatility forecasts through the effect of volatility trading strategies. Taking into account all these effects, the candidate model we propose is

\[
\log (RV)_{t,t+h} = \alpha + \beta^+ \cdot \log (CSV^+) + \beta^- \cdot \log (CSV^-) + \\
\gamma^+ \cdot \log (1 + JSV^+) + \gamma^- \cdot \log (1 + JSV^-) + \\
\theta \cdot \log (1 + \text{VolJ}) + \delta \cdot cRet^- + \delta_w \cdot cRet_{(t-5,t]}^- + \\
\vartheta_w \cdot \log (RV)_{(t-5,t]} + \vartheta_m \cdot \log (RV)_{(t-22,t]} + error_t, \quad (1.29)
\]

where past negative returns and jumps are defined by \(cRet^- = \mathbb{1}_{\{cRet < 0\}} \cdot cRet\), and \(cRet_{(t-5,t]}^- = \frac{1}{5} \sum_{i=t-4}^t cRet_i^-\). Note that the realized jump semivariances already capture the effect of return jumps, since they are derived from them. The interpretation of their effects is that of jump risks that capture jump leverage effects. Therefore, return jumps are not included in this model specification.

Given that one of the goals of this study is volatility forecasting, one can argue that such a model may be overparametrized and therefore may lead to poor out-of-sample results. As we will show in the next sections this is not (entirely) the case. Nevertheless, to overcome this problem we consider a simplified version of model (1.29) that differs from the previous one by discarding jump semivariances, the weekly persistency in the leverage effect,
and the jumps in volatility\(^3\). Moreover the impact of jump semivariations on future volatility are being replaced by that of return jumps which is to be interpreted as a leverage effect due to jumps:

\[
\log (RV)_{t,t+h} = \alpha + \beta^+ \cdot \log (CSV^+)_{t} + \beta^- \cdot \log (CSV^-)_{t} + \delta \cdot cRet_t^- + \varphi \cdot jRet_t + \vartheta_w \cdot \log (RV)_{(t-5,t]} + \vartheta_m \cdot \log (RV)_{(t-22,t]} + error_t.
\] (1.30)

This model is very simple, consisting only of seven parameters to be estimated, e.g., three parameters less than the \(LHAR - CJ\) model proposed by Corsi and Renò (2010). Still, the model focuses on the most relevant effects for volatility forecasting: downside risk, leverage effect, and the \(HAR\) structure capturing long-memory.

The models are estimated by \(OLS\) with Newey and West (1987) covariance matrix correction to account for serial correlation. The bandwidth used is \(2(h - 1)\), where \(h\) is the forecasting horizon.

### 1.4 Empirical Evidence

#### 1.4.1 Leverage Effect, Volatility Feedback Effect, and Persistence

Leverage and volatility feedback effects are commonly studied by mean of correlations. Bollerslev \textit{et al.} (2006) provide exhaustive evidence of the leverage effect at high frequency by studying cross-correlations among return and volatility series for a horizon spanning several days. Exploiting the methodology we use to disentangle the continuous and jump components of return and volatility dynamics, we present new evidence of leverage and volatility feedback effects arising from both continuous and jump components. Moreover, we use signed intra-day returns to capture sign asym-

\(^3\)We tried to discard these effects one-by-one with an approach similar to backward stepwise subset selection and verified that the results both in-sample and out-of-sample were quantitatively similar, yielding the specification (1.30).
1.4. Empirical Evidence

metrics. Figure 1.4 reports cross-correlations among different components of the return and quadratic variation as evidence of leverage and volatility feedback effects, and persistency in volatility.

As in Bollerslev et al. (2006), the leverage effect (negative correlation between $RV_{t+h}$ and $cRet_t$) is significant for prolonged days, while there is no evidence of a negative correlation between $RV_{t-h}$ and $cRet_t$). The latter seems to be even positive. The sign and size asymmetry of the leverage effect becomes evident as negative returns generate all the negative correlation of return and lagged volatility (see top right plot, Figure 1.4) and the magnitude of this effect is higher than the positive correlation between positive returns and realized variation. The negative correlation between future realized variation and present negative returns is rather small and insufficient to generate the negative size asymmetry for the volatility feedback effect.

Jumps in return are highly responsible for the leverage effect. The negative correlation between jumps and future volatility is very high in magnitude (see plots on the second row of Figure 1.4) but this effect is short lived, lasting only one day period. The plots on the third and fourth rows of Figure 1.4 reports the cross correlation of realized variation with signed continuous and jump semivariations. All those components have an impact on future volatility and they represent different sources of risk.

The persistence in volatility is also examined. It is well known that volatility is autocorrelated for a prolonged period of time. Evidence is found in Ding et al. (1993) for the S&P500 stock index. Models to capture the persistence in conditional volatility have been proposed in their early stage by Engle and Bollerslev (1986) and Baillie et al. (1996). This autocorrelation is one of the drivers of the volatility feedback effect. As mentioned above, when an increase in volatility is associated with an expectation of higher future volatility, market participants may discount this information, resulting in an immediate drop in stock prices. We find that the persistence in volatility is generated by the continuous component of the return variation.
Clearly, jump variation has a confounding effect on realized variation as well, but this effect is also short lived (The cross-correlations of realized variation with continuous variation and with jump variation are reported in the plots on the fifth row of Figure 1.4).

Finally, both continuous return and jumps in return are negatively and contemporaneously correlated with volatility jumps. Moreover, they have a small and short-lived impact on future volatility jumps. On the other hand, volatility jumps also have a short-lived impact only on future return jumps and continuous returns (Cross-correlations of continuous returns and return jumps with jumps in volatility, depicting asymmetries for the jump components of the volatility, are represented in the last row of Figure 1.4).

1.4.2 In-sample Analysis

Estimation results of the proposed forecasting models are discussed in this section. The in-sample evaluations are based on different horizons. The regression results of the candidate models are reported in Table 1.2, for horizons of 1, 5, 15 and 22 days.

In order to check the stability of the coefficients, regressions are run for different overlapping subsamples. As the full sample period is relatively long, consisting of more than 28 years, it is likely that structural breaks have occurred. This seems to be the case, as some coefficients estimated by including data for the initial 8 years, from 1982 to 1989 (the forecasts are relative to the period 1990-1997), appear to differ slightly from those estimated by using data starting from 1990. Conversely, the coefficients associated with the model for the last 20 years are relatively stable. Figure 1.5 reports the estimated coefficients of one period forecast of model (1.29) with rolling subsamples of 2000 observations (corresponding to almost 8 years).

With an in-deep inspection, the difference of the coefficient estimates for the initial subsamples from those of the remaining subsamples is mainly caused by the market crash of October 1987. The sample period is there-
fore reduced and the results of the in-sample analysis contained in Table 1.2 are relative to the subperiod from January 4, 1988 to August 6, 2010. By contrast, the out-of-sample evaluations of the next sections, as they are performed using rolling forecasts, will be based on the whole sample period. Volatility jumps have a marginal power to forecast the one period ahead volatility only with the initial subsamples, which contain the market crash data, while for the remaining subsamples they do not add further value. This indicates that volatility jumps are useful for forecasting during extremely agitated periods.

To check the stability of the estimates for different forecasting horizons, Figure 1.6 reports the estimated coefficients for forecasting horizons ranging from 1 day to 30 days. The estimated parameters appear to be well-behaved. The 95% forecast confidence intervals (adjusted with Newey-West serial correlation consistent standard errors), are relatively large for the coefficients associated with jump semivariations.

The following results from the estimation are worth remarking on. “Bad volatilities” have a significant impact on future volatility. Both continuous and jump downside semivariations increase future realized variation. However, when “good volatilities” are disentangled into upside continuous variation and upside jump variation, a remarkable difference emerges. The effect of upside continuous semivariation on future volatility is still positive and statistically significant, but that of the upside jump semivariation is on average negative and negligible. This may well answer the critique advanced by Corsi et al. (2010) as they argue that influential studies such as Andersen et al. (2007a) and Forsberg and Ghysels (2007), among others, find a negative or null impact of jumps (more appropriately jump variation) on future volatility, while economic theory suggests the opposite. In fact, one needs to distinguish between downside and upside jumps. Future

---

4Although the estimated coefficients obtained by using the full sample data, including the market crash of 1987, do not differ substantially from the ones reported in Table 1.2, there may be a minor loss in terms of consistency of the estimates by using the full sample data. However, the signs associated with all significant coefficients are the same for the various subsamples.
volatility is indeed increasing with downside jump variation as jump variations are likely associated with an increase in uncertainty on fundamental values. However the effect of upside jump variation does not necessarily increase future volatility. This would be consistent with the economic model of Veronesi (1999). Intuitively, a positive jump is associated with the occurrence of good news as well as the expectation of higher future returns. The effect of the latter is able to offset the effect of an increasing uncertainty. The future volatility given the occurrence of a positive jump is therefore lower than the one associated with the occurrence of a negative jump.

Consistent with the leverage effect, both continuous and jump components of the return have a significant impact on future volatility. As mentioned previously, the persistency in leverage effect is captured mainly by the continuous component. In fact, as the forecasting horizon increases, the leverage effect generated by past jumps becomes less statistically significant. On the contrary, the coefficient associated with the negative continuous return over the past day and over the past week remains statistically significant even for longer forecasting horizons.

Concerning long memory features, the persistence parameter associated with the one week and one month realized volatility is highly statistically significant for all horizons. Ultimately, jumps in volatility (continuous variation) do not appear to be statistically significant. Given that volatility jumps are present during extremely agitated periods, their null forecasting performance on the short horizon is due to the fact that only a few volatility jumps are identified for the sample period under investigation. There are in fact volatility jumps for only 3.6% of the sample days.

Finally, by examining the (in-sample) performance of the simplified model (1.30) in comparison with the one of the full model, the losses in accuracy that occur for the simplified model are negligible based on the adjusted $R^2$. 
1.4.3 Out-of-Sample Forecasting Performance

The methodology used to assess the forecasting performance of the proposed models is presented in this section. The analysis is based on the out-of-sample predictive accuracy of these models in comparison to the HAR–RV model (1.27). The predicted variable is the cumulative average log realized variation between time $t$ and $t + h$. For predictions of horizons $h > 1$, a “direct method” is employed, that is, the model specifies only the relation between $\log(RV)_{t,t+h}$ and the regressors at time $t$. The evaluation of the performance is done recursively with rolling windows. The forecasting horizons considered are $h = \{1, 5, 15, 22\}$.

The recursive procedure is applied as follows: The forecasts are generated by using an in-sample estimation window of 2000 observations, corresponding to about eight years, starting from April 28, 1982. For $h = 1$, the performance is evaluated on 5040 out-of-sample data points, corresponding to about 20 years. The forecasting performance is based on the $MSE$ function of the log realized variation forecasts and the negative $QLIKE$ loss function:

\[ MSE = \left( \ln(\widehat{RV}_{t,t+h}^{t+h} | \mathcal{F}_t) - \ln(RV)_{t,t+h} \right)^2, \]
\[ QLIKE = \ln(\widehat{RV}_{t,t+h}^{t+h} | \mathcal{F}_t) + \frac{RV_{t,t+h}}{RV_{t,t+h}^{t+t+h} | \mathcal{F}_t}. \]

The $MSE$ is a symmetric loss function while $QLIKE$ is asymmetric. Patton (2011) shows that the $QLIKE$ loss function is robust to noise in the volatility proxy, as volatility forecasts represent a case where the true values are not observable. Moreover, this function has certain optimal properties in that it is less sensitive to large observations by more heavily penalizing under-predictions than over-predictions.

To test for the superior forecasting performance of the proposed models over the benchmark $HAR – RV$ model, the Diebold and Mariano (1995) test is employed. The asymptotic variance of the loss differential (of each model
and the $HAR - RV$ model) is estimated with Newey-West $HAC$ consistent sample variance as suggested by Giacomini and White (2006). Clark and McCracken (2001) show that in the presence of nested models, the distribution of the Diebold-Mariano test statistics for mean square error losses, under the null, can be nonstandard. Therefore the test of forecasting performance for mean square error losses is based on the Clark and West (2007) test statistic, which appropriately corrects the loss differential. Table 1.3 reports the test statistics under the null of equal forecasting performance for each model pair. A positive loss differential represents a superior average forecasting performance of the proposed model with respect to the $HAR - RV$.

At the 99% confidence level, the proposed models outperform the base model for forecasting horizons of one day and five days. For longer forecasting horizons, overall the models also perform well, with the reduced model (equation 1.30) achieving surprisingly high performance. Based on the $QLIKE$ loss function, the outperformance of the full model (1.29) compared with the $HAR - RV$ model is only weakly statistically significant. This is a warning signal that model (1.29) is probably overparametrized, although as shown in the next section, outperforming the benchmark $HAR - RV$ model in forecasting long-run volatility seems to be a difficult task.

### 1.4.4 Model Confidence Set

The $HAR$ model in logarithmic form generally has a good out-of-sample forecasting performance. In an attempt to raise the bar, other reference models recently proposed in the literature are considered. First, we test the performance of the $HAR$ model for the realized variation, which yielded the lowest forecasting power among all the candidate models when evaluated on the loss functions (2.39) and (2.40). Then, we investigate the accuracy of the volatility forecasts for several models involving two model specifications of Andersen et al. (2007a), named $HAR - RV - J$ and $HAR - RV - CJ$ (eq. 13, p. 709, and eq. 28, p. 715), which explicitly take into account con-
tinuous and jump variation; two models proposed by Patton and Sheppard (2011) based on semivariances, one with complete decomposition into semivariances and the other with a downside semivariance only for the daily component (eq. 19, p. 16, and eq. 17, p. 13, without upside semivariance); and two models of Corsi and Renò (2010) that add the leverage effect, named \( \text{LHAR} - CJ \) and \( \text{LHAR} - CJ^+ \) (eq. 2.4, p. 8, and table 3, p. 16). Finally, simple and commonly used models are also taken into account. These are an autoregressive model on the daily component only and an exponential smoothing model on log realized variation. The last is popularly used by risk practitioners (see Taylor, 2004) and is specified as

\[
\hat{\log}(RV)_{t,h} = \alpha \cdot \log(RV)_{t-h} + (1 - \alpha) \cdot \log(RV)_{h,t}
\]

(1.33)

with parameter \( \alpha \) optimized for each rolling window following

\[
\arg \min_{\alpha} \sum_t \left[ \log(RV)_{t-h} - \log(RV)_{h,t} \right].
\]

All the models, except the \( \text{HAR} - RV \), are evaluated on log realized variation. They are also estimated on the same rolling window and evaluated on the same out-of-sample data points. It is also worth mentioning that jumps detection and jump variation estimation are executed in the same way as described here.\(^5\)

In order to evaluate forecasts of those models, the model confidence set (MCS) methodology of Hansen et al. (2003, 2011) is the most well-suited. The comparison is done among a set of models, as pairwise comparisons would not be appropriate. The methodology allows the models to be ranked based on a given loss function and it gives indication of whether the forecast

\(^5\)In their original work, Andersen et al. (2007a) do not apply a test for significant jump detection but jumps are identified as positive differences between \( RV \) and \( BV \). Corsi and Renò (2010) apply a test based on the difference between \( RV \) and a threshold estimator for the continuous variation. Numerically, the results may differ slightly with different jump identification procedures. However, the performance of our proposed models is robust even compared to other testing procedures for jumps.
performances are significantly different. Out of the surviving models in the confidence set, the interpretation is that they have equal predictive ability and they yield the best forecasts given a confidence level.

The model confidence set approach allows the user to specify different criteria to establish equal forecasting performance for the model set and subsets. We use both the “range” statistic, and the “semi-quadratic” statistic:

\[
T_R = \max_{k,s \in \text{Mset}} \frac{|\hat{d}_{k,s}|}{\sqrt{\hat{\text{var}}(\hat{d}_{k,s})}},
\]

\[
T_{SQ} = \sum_{k,s \in \text{Mset}} \frac{\hat{d}_{k,s}^2}{\hat{\text{var}}(\hat{d}_{k,s})},
\]

where \( \hat{d}_{k,s} \) is the mean loss differential between each pair combination of models, with \( k \) and \( s \) denoting each model.

Table 1.4 reports the model confidence set and the selected models at the 5% and 15% significance levels. The model confidence set p-value is obtained through a block bootstrap procedure. An autoregressive process is estimated for each \( d_{k,s} \), the loss differential between each model \( k \) and \( s \), and the lag length for it is determined by Akaike information criteria as suggested by Hansen et al. (2003). The block length for the bootstrap procedure is then fixed as the maximum lag length among \( d_{k,s} \) and it varies between 9 and 30 depending on the forecasting horizon and the loss function used. 5000 bootstrap repetitions are used to compute the test statistics. The model confidence set p-value obtained by using the semi-quadratic test statistics is less conservative than the one obtained by using the range statistics. The surviving model set after using the semi-quadratic test statistic is therefore larger than the set obtained by using the range test statistic.

Overall, the best forecasting models for all different horizons are the candidate models (1.29) and (1.30) and the two \( LHAR - CJ \) models proposed by Corsi and Renò (2010), which take into account past negative return without disentangling return jumps size. The reduced model of equation (1.30)
achieved equal forecasting performance, compared with the $LHAR - CJ^+$ model, with a lower model complexity (4 variables less). Jumps in return indeed substantially improve the forecasting performance, while jumps in continuous variation (called volatility jumps) are infrequent and have a minor effect on future realized variation. Jumps in quadratic variation are in fact mainly due to return jumps. For the one period ahead forecast, the full model (1.29), which includes volatility jumps, does not survive based on the $MSE$ loss function. However, it does survive based on the $QLIKE$ loss function. This is due to the fact that by adding volatility jumps, the model tends to produce marginally less conservative forecasts of volatility. The $QLIKE$ loss, since it less heavily penalizes over-predictions than under-predictions, still selects this model. Not surprisingly, the best forecasting models are the ones that include the leverage effect.

Finally, the following considerations can be pointed out for the simplest models: For long horizons, the simple $HAR - RV$ model is hard to beat. As has already been shown previously in the literature, the autoregressive model that takes only the daily component into account is not selected for any forecasting horizon and loss function used, pointing to the importance of correctly modeling long-memory. Moreover, the exponential smoothing model, often used in practice, is clearly outperformed.

1.5 Conclusion

This chapter analyzed the performance of volatility forecasting models that take into account downside risk, jumps, and the leverage effect. The volatility forecasting model proposed consists of the following ingredients: First, the size of return jumps is identified based on the test of Lee and Mykland (2008). Second, jump variation and continuous variation are disentangled based on the methodology proposed by Andersen et al. (2010). Third, the size of jumps in volatility is also identified with inference based on an auxiliary $AR - GARCH$ model. Finally, signed jump and continuous semivari-
lations are computed using signed intra-day returns. The best candidate model for forecasting realized volatilities must simultaneously take into account return jumps, “good” and “bad” risks, leverage effect, and strong volatility persistence (i.e. long memory).

The model is motivated by overwhelming evidence of asymmetries in financial time series. We show that correlation asymmetries are present for both continuous and jump components among return and volatility processes. Moreover, asymmetries exist not only in size but also in sign, justifying the use of semivariances in forecasting volatility.

The forecasting model is very simple to implement as based on the parsimonious HAR framework. The gain over the base HAR – RV in terms of out-of-sample forecasting power is substantial and this is especially true for short and mid forecasting horizons. Therefore, volatility forecasts with return and jump asymmetries are warranted.
Table 1.1: Jump Descriptive Statistics

<table>
<thead>
<tr>
<th>Year</th>
<th>n. Days with Return Jump</th>
<th>% Days with Return Jump</th>
<th>Mean Jump Intensity (One Day)</th>
<th>Max Jump Intensity (One Day)</th>
<th>% Days with Volatility Jump</th>
</tr>
</thead>
<tbody>
<tr>
<td>1982*</td>
<td>31</td>
<td>17.92</td>
<td>1.10</td>
<td>3</td>
<td>6.94</td>
</tr>
<tr>
<td>1983</td>
<td>29</td>
<td>11.51</td>
<td>1.38</td>
<td>6</td>
<td>1.98</td>
</tr>
<tr>
<td>1984</td>
<td>48</td>
<td>19.05</td>
<td>1.13</td>
<td>2</td>
<td>3.57</td>
</tr>
<tr>
<td>1985</td>
<td>50</td>
<td>19.76</td>
<td>1.22</td>
<td>4</td>
<td>0.40</td>
</tr>
<tr>
<td>1986</td>
<td>48</td>
<td>18.97</td>
<td>1.60</td>
<td>7</td>
<td>5.14</td>
</tr>
<tr>
<td>1987</td>
<td>36</td>
<td>14.63</td>
<td>1.89</td>
<td>10</td>
<td>5.28</td>
</tr>
<tr>
<td>1988</td>
<td>45</td>
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<td>1.47</td>
<td>6</td>
<td>3.97</td>
</tr>
<tr>
<td>1989</td>
<td>42</td>
<td>16.67</td>
<td>1.48</td>
<td>8</td>
<td>2.38</td>
</tr>
<tr>
<td>1990</td>
<td>47</td>
<td>18.65</td>
<td>1.51</td>
<td>5</td>
<td>3.17</td>
</tr>
<tr>
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<td>8</td>
<td>1.20</td>
</tr>
<tr>
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<td>6</td>
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<tr>
<td>1994</td>
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<td>5</td>
<td>1.59</td>
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<td>1995</td>
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<td>19.2</td>
<td>1.38</td>
<td>4</td>
<td>0.40</td>
</tr>
<tr>
<td>1996</td>
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<td>19.6</td>
<td>1.49</td>
<td>5</td>
<td>4.40</td>
</tr>
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<td>1.77</td>
<td>6</td>
<td>7.26</td>
</tr>
<tr>
<td>1998</td>
<td>48</td>
<td>19.2</td>
<td>1.33</td>
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<td>5.20</td>
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<tr>
<td>1999</td>
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<tr>
<td>2000</td>
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<tr>
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<td>2002</td>
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<td>13.31</td>
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<tr>
<td>2003</td>
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<td>2004</td>
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<td>4</td>
<td>1.23</td>
</tr>
<tr>
<td>2005</td>
<td>67</td>
<td>27.13</td>
<td>1.33</td>
<td>4</td>
<td>0.00</td>
</tr>
<tr>
<td>2006</td>
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<tr>
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<td>1.65</td>
<td>7</td>
<td>3.63</td>
</tr>
<tr>
<td>2008</td>
<td>48</td>
<td>19.43</td>
<td>1.38</td>
<td>4</td>
<td>7.29</td>
</tr>
<tr>
<td>2009</td>
<td>44</td>
<td>17.6</td>
<td>1.41</td>
<td>5</td>
<td>4.80</td>
</tr>
<tr>
<td>2010**</td>
<td>35</td>
<td>23.33</td>
<td>1.49</td>
<td>4</td>
<td>4.67</td>
</tr>
<tr>
<td>Full Sample</td>
<td>19.29</td>
<td>1.43</td>
<td>3.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: The table reports the number or days with return jumps, their percentage, the average daily jump intensity conditional on the presence of at least one jump, the maximum daily jump intensity, and the percentage of days with jumps in continuous variation. All the statistics are sorted by year. The tests for jumps in return and in continuous variation are conducted with significance level $\alpha = 0.01$. *The sample period for 1982 starts on April, 28. **The sample period for 2010 stops on August, 6.
Figure 1.3: Jump Contributions

NOTE: The figure plots the average contributions of positive jumps to positive returns, negative jumps to negative returns, jump variation to realized variation and volatility jumps to continuous variation. The percentages are calculated by using rolling windows consisting of three months and one year.
1.5. Conclusion

Figure 1.4: Sample Cross-correlation Between Realized Variation, Return, and Jumps

NOTE: The figure plots pairwise sample cross-correlation of daily realized variations with daily returns and return jumps (first row), signed returns and jumps (second and third rows), continuous and jump variations (fourth row), and the cross-correlation between volatility jumps and continuous and jump returns (last row). Lags and leads of up to 22 days are considered.
### Table 1.2: In-sample Regressions

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Model</th>
<th>Associated Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Cons</td>
</tr>
<tr>
<td>1 day</td>
<td>1</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[1.16]</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[4.89]</td>
</tr>
<tr>
<td>5 days</td>
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<td>-0.004</td>
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<tr>
<td></td>
<td></td>
<td>[-0.14]</td>
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<tr>
<td></td>
<td>2</td>
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<tr>
<td></td>
<td></td>
<td>[2.27]</td>
</tr>
<tr>
<td>15 days</td>
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<td>-0.064</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td></td>
<td>[-0.35]</td>
</tr>
<tr>
<td>22 days</td>
<td>1</td>
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<tr>
<td></td>
<td></td>
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<td></td>
<td>2</td>
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<tr>
<td></td>
<td></td>
<td>[-1.05]</td>
</tr>
</tbody>
</table>

**NOTE:** The table contains estimates of the coefficients and t-statistics, in square brackets, based on Newey-West HAC consistent standard errors, for the two model specifications. Model 1 and model 2 correspond to equations (1.29) and (1.30), respectively. The models are estimated for forecasting horizons of 1, 5, 15 and 22 days.
Figure 1.5: Rolling Window Coefficients for the One Period Ahead Forecast

NOTE: The figure plots rolling window coefficient estimates with the corresponding 95% confidence interval, based on Newey-West HAC consistent standard errors. The coefficients are the ones associated with the model in equation (1.29) for the 1-day ahead forecast. The estimation window size consists of 2000 observations.
Figure 1.6: Parameter Estimates with Respect to Different Forecasting Horizons

NOTE: The figure plots the estimated coefficients with the corresponding 95% confidence interval, based on Newey-West HAC consistent standard errors, for the model in equation (1.29). The forecasting horizon, on the horizontal axis, ranges from 1 to 30 days.
1.5. Conclusion

Table 1.3: Out-of-Sample Forecast Test

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>1 day</th>
<th>5 days</th>
<th>15 days</th>
<th>22 days</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM Test Statistics</td>
<td>1</td>
<td>4.87</td>
<td>4.34</td>
<td>2.38</td>
<td>1.80</td>
</tr>
<tr>
<td>with QLIKE Losses</td>
<td>2</td>
<td>4.44</td>
<td>4.65</td>
<td>3.16</td>
<td>3.13</td>
</tr>
<tr>
<td>CW Test Statistics</td>
<td>1</td>
<td>11.34</td>
<td>10.83</td>
<td>7.72</td>
<td>6.67</td>
</tr>
<tr>
<td>with MSE Losses</td>
<td>2</td>
<td>11.70</td>
<td>10.84</td>
<td>7.73</td>
<td>6.79</td>
</tr>
</tbody>
</table>

NOTE: The table reports results from the out-of-sample pairwise forecast performance comparison. The performance is based on the QLIKE and MSE loss functions for models 1 and 2 corresponding to equations (1.29) and (1.30), respectively. Each model is evaluated against the benchmark HAR – RV (equation 1.27). Forecast horizons considered are 1 day, 5 days, 15 days, and 22 days. The upper panel reports the Diebold-Mariano t-statistics on QLIKE losses and the bottom panel reports the Clark-West test statistics on MSE losses. The out-of-sample performance period ranges from May 1990 to August 2010.
### Table 1.4: Model Confidence Set

<table>
<thead>
<tr>
<th>Model</th>
<th>Statistics</th>
<th>QLIKE Loss Function</th>
<th>MSE Loss Function (on log RV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Forecasting Horizon</td>
<td>1 Day</td>
<td>5 Days</td>
</tr>
<tr>
<td>HAR (log RV)</td>
<td>Mean loss (Full Sample)</td>
<td>0.524</td>
<td>0.453</td>
</tr>
<tr>
<td></td>
<td>MCS (R-Stat)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCS (SQ-Stat)</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>HAR (RV)</td>
<td>Mean loss (Full Sample)</td>
<td>0.614</td>
<td>0.682</td>
</tr>
<tr>
<td></td>
<td>MCS (R-Stat)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>MCS (SQ-Stat)</td>
<td>+</td>
<td>++</td>
</tr>
<tr>
<td>Mod. 1</td>
<td>Mean loss (Full Sample)</td>
<td>0.514</td>
<td>0.449</td>
</tr>
<tr>
<td></td>
<td>MCS (R-Stat)</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td></td>
<td>MCS (SQ-Stat)</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>Mod. 2</td>
<td>Mean loss (Full Sample)</td>
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<td>0.449</td>
</tr>
<tr>
<td></td>
<td>MCS (R-Stat)</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
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<td>MCS (SQ-Stat)</td>
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<td>++</td>
</tr>
<tr>
<td>Semivariances - HAR 1</td>
<td>Mean loss (Full Sample)</td>
<td>0.521</td>
<td>0.452</td>
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<tr>
<td>(PS, 2011)</td>
<td>MCS (R-Stat)</td>
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<td>MCS (SQ-Stat)</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>Semivariances - HAR 2</td>
<td>Mean loss (Full Sample)</td>
<td>0.521</td>
<td>0.452</td>
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<tr>
<td>(PS, 2011)</td>
<td>MCS (R-Stat)</td>
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</tr>
<tr>
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<td>MCS (SQ-Stat)</td>
<td>++</td>
<td>++</td>
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<tr>
<td>HAR-RV-I (ABD, 2007)</td>
<td>Mean loss (Full Sample)</td>
<td>0.524</td>
<td>0.453</td>
</tr>
<tr>
<td></td>
<td>MCS (R-Stat)</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>MCS (SQ-Stat)</td>
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<tr>
<td>HAR-RV-CJ (ABD, 2007)</td>
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<td>0.524</td>
<td>0.452</td>
</tr>
<tr>
<td></td>
<td>MCS (R-Stat)</td>
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<td>MCS (SQ-Stat)</td>
<td>++</td>
<td></td>
</tr>
<tr>
<td>LHAR-CJ (CR, 2010)</td>
<td>Mean loss (Full Sample)</td>
<td>0.513</td>
<td>0.449</td>
</tr>
<tr>
<td></td>
<td>MCS (R-Stat)</td>
<td>++</td>
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<tr>
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<td>MCS (SQ-Stat)</td>
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<td>++</td>
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<tr>
<td>LHAR-CJ+ (CR, 2010)</td>
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</tr>
<tr>
<td></td>
<td>MCS (R-Stat)</td>
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<tr>
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<td>MCS (SQ-Stat)</td>
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<td>++</td>
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<tr>
<td>Autoregressive 1 day</td>
<td>Mean loss (Full Sample)</td>
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<tr>
<td>Exponential Smoothing</td>
<td>Mean loss (Full Sample)</td>
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<td></td>
<td>MCS (R-Stat)</td>
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</tr>
<tr>
<td></td>
<td>MCS (SQ-Stat)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**NOTE:** The table reports the forecasting performance of models in the confidence set. A block bootstrap procedure with 5000 replications is used to establish equal predictive ability of the surviving models according to both “range” and “semi-quadratic” test statistics. Moreover, both QLIKE and MSE (on log realized variation) losses are used. Forecast horizons considered are 1 day, 5 days, 15 days, and 22 days. “+” denotes included models in the final confidence set with significance level 5%, and “++” denotes included models in the confidence set with significance level 15%, with $\text{MCS}_{15\%} \subseteq \text{MCS}_{5\%}$. Mod. 1 and Mod. 2 correspond to equations (1.29) and (1.30), respectively.
2.1 Introduction

A source of risk that global investors and companies face is the fluctuation of foreign currency rates. With recent advancements in financial econometrics, this risk can be measured with the high frequency data based realized covariance. However, the use of such a measure for the covariance matrix as a proxy of financial risk challenges the adequacy of low frequency data based models to forecast financial volatility. Two questions immediately arise: If the ex-post true conditional volatility is the realized volatility, would the use of low frequency based return information be inconsistent ex-ante? Is there any economic gain in using high frequency data to forecast the conditional covariance matrix of exchange rate returns?

In this chapter, I examine the accuracy of several multivariate models to forecast exchange rate volatility under the assumption that the true ex-post covariance matrix is the realized covariance of Barndorff-Nielsen and Shephard (2004a). The proposed models are all based on the statistical decompo-
sition of the conditional covariance matrix in terms of conditional standard deviations and the conditional correlations, in a similar fashion as in the dynamic conditional correlation model (DCC) of Engle (2002). Thus, those models are all similar in nature but differ among them for the frequency of the data used.

An economic evaluation criterion is implemented in order to assess the economic significance of the different model forecasts. A volatility timing strategy is carried out in forming portfolios of currencies based on the forecast of their covariance matrix. The performance of those portfolios is assessed in terms of their ex-post realized volatility. The volatility timing strategy on FX returns takes the perspective of a fund manager who is constraint to invest in foreign currencies. All the volatility models are also assessed based on their density forecast ability and their adequacy for the management of the tail risk with a Value-at-Risk backtesting.

Forecasting multivariate volatility is subject of an intensive literature for more than two decades. For low frequency GARCH-type models, Bauwens et al. (2006) and Silvennoinen and Teräsvirta (2009) offer an excellent review of the literature. The modeling of the multivariate volatility, i.e., the covariance matrix, with high frequency data is a recent topic of research. Flexible models can be constructed based on the Wishart autoregressive (WAR) process of Gourieroux et al. (2009) that guarantees positive definitiveness of the predicted covariance matrix. In addition to this, other methods have been proposed. Chiriac and Voev (2011) introduce a methodology to forecast the covariance matrix based on the Cholesky decomposition of the realized covariance. They show that a vector autoregressive fractionally integrated moving average (VARFIMA) process can offer a high performance in forecasting when applied to the Cholesky matrix. Instead, Bauer and Vorkink (2011) rely on the matrix log transformation of the covariance matrix that guarantees positive definitiveness via subsequent matrix exponential transformation of the predicted matrix. Despite those methods offer a clever solution to ensure positive definitiveness, the difficulty to interpret
their parameter estimates remains a problem. Conscious of this problem a new class of models that takes high frequency return information but still being based on the multivariate GARCH framework have been proposed. Noureldin et al. (2011) introduce a multivariate mixed model in terms of return data frequency used based on the BEKK-type parametrization of Engle and Kroner (1995). This extends previous work in the univariate framework by Engle and Gallo (2006), Shephard and Sheppard (2010) and Hansen et al. (2010). Research on mixed frequency models is at its infancy.

Instead of proposing a completely new class of model, I investigate how simple models, whether they take information at low frequency, high frequency or mixed frequency, perform in terms of out-of-sample forecasting power. An examination of this issue has been carried out by Halbleib and Voev (2011). In their analysis, a mixed frequency model for the covariance, resulting from the composition of high frequency based realized volatility forecasts and low frequency based correlation forecast performs relatively well and may be preferable in agitated periods. Contrary to their finding, my research points out that to forecast realized covariance high frequency correlation information should be taken into account. Moreover, in order to maintain comparability of the forecasting models, I estimate the conditional latent volatility with GARCH-type models and do not assume that current conditional volatility is observable as the realized volatility. The latter is observed only ex-post. In the univariate framework, Giot and Laurent (2004) compare the performance of a GARCH-type model with a high frequency ARFIMAX model, that includes the realized volatility in the conditional variance of the returns process, in order to forecast daily Value-at-Risk. My analysis provides evidences broadly consistent with their paper, despite I use a GARCHX model for the latent volatility with realized volatility information. In particular, daily portfolio returns standardized by the 1-day-ahead forecasts of its volatility (i.e. generalized volatility) depart from the normal law.

A strand of the literature on volatility modeling aims to examine the
model implied economic value. It is often the case that statistical models are questioned on their effectiveness to generate a value added. Fleming et al. (2001) propose to analyze the performance of asset portfolios built upon estimates of volatilities. They find that a volatility timing investment strategy indeed leads to a higher economic profit compared to an investment strategy based on a static volatility. Bandi et al. (2008) specifically evaluate this economic gain in terms of the optimal high frequency sampling frequency for the realized covariance. They find that even within high frequency sampling, one needs to estimate covariances with the optimal procedure proposed by Bandi and Russell (2008) in order to achieve a higher economic value, given the volatility timing strategy. The economic evaluation carried out in this paper relates to the out-of-sample forecast of the covariance matrix. The analysis confirm the usefulness of high frequency correlation information in order to achieve the goal of minimizing portfolio realized variance.

The motivation of this chapter stems from the popularity in financial econometrics of using the realized measure as the true ex-post observed measure for multivariate volatility. In the univariate framework Hansen and Lunde (2006a) advocate the use of a realized measure as proxy for the latent true volatility and this is supported by the fact that the high sampling density makes the realized measure a more accurate proxy for the latent volatility with respect to the traditional squared daily return. Given that the true ex-post covariance matrix is based on high frequency data, the intuition call for a forecasting model that include such a high frequency measure in order to increase the forecasting accuracy. The empirical literature is silent about this point. It is yet unclear what is the gain of using higher frequency information in an out-of-sample forecasting perspective, if there is such a gain. To address this issue the forecasting models needs to be as similar as possible, with the only difference being the sampling frequency.

This chapter is structured as follows. Section 2.2 describes the forecasting models for the conditional covariance matrix arising from the its decompo-
sition in conditional standard deviations and correlations. The evaluation of covariance forecasts with exchange rate data is carried out within Section 2.3. Section 2.4 explains the portfolio realized volatility evaluation methodology and presents the related results. The evaluation of density forecast the Value-at-Risk backtesting is contained in Section 2.5. Finally, Section 2.6 concludes.

2.2 Multivariate Volatility Models

As the focus of this chapter is in the out-of-sample forecasting performance, the models are selected to be as simple as possible without undue parametrization. Thus, they do not include jumps, volatility feedback and spill-over effects. In order to model spill-over effects the complexity of the models would need to increase substantially without clear benefits in terms of the out-of-sample performance. As the application presented here concerns FX markets, volatility feedback (or leverage) effects would not find an economic rationale. As for jumps, Boudt et al. (2011) argues that jumps in multivariate volatility models should be considered outliers as their inclusion leads often to an overestimation of future volatility. They propose weighting functions to smooth the impact of past shocks. Their results indicate that a jump robust DCC model significantly outperforms the traditional DCC model, especially in calm periods. As this chapter circumscribes the volatility forecasting problem within the sampling frequency of return information used, the jump component is not modeled separately. Return jumps are inevitably identified differently depending on the sampling frequency.

All the models analyzed are based on the following decomposition. Denote with $S_t$ the vector of returns with $k$ elements and with $\Sigma_t$ the $(k \times k)$ covariance matrix at time $t$, the multivariate stochastic return process is rep-
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represented by

\[ S_t = D_{t|t-1} \varepsilon_t, \quad (2.1) \]

\[ \Sigma_{t|t-1} = D_{t|t-1} R_{t|t-1} D_{t|t-1}, \quad (2.2) \]

where \( D_{t|t-1} \equiv \text{diag} \left[ h_{1,1|t-1}^{1/2}, \ldots, h_{k,k|t-1}^{1/2} \right] \) is a matrix containing conditional volatilities on the main diagonal and zero elsewhere and \( R_{t|t-1} \) is the conditional correlation matrix. The conditional mean of \( S_t \) is set to zero.

Five forecasting models for the covariance matrix \( \Sigma_t \) are conceived depending on the forecast of conditional volatilities \( D_t \) and correlations \( R_t \). All the models are estimated with the two step quasi-maximum likelihood procedure propose by Engle and Sheppard (2001) and they are all ensured to produce positive definite forecasts of the conditional covariance matrix.

2.2.1 Conditional Volatility

A GARCH structure is used to model the latent conditional volatility. The modeling strategy consists in discriminating forecasts of volatility by imposing a specific proxy for the past (conditional) volatility. In case the model uses only low frequency (daily) information, the (daily) squared return represents the proxy for the latent variance. The GARCH(1,1) model for each individual return series \( r_{i,t} \), with \( i = 1, \ldots k \) and \( t = 1, \ldots T \), reads as

\[ r_{i,t} = \sqrt{h_{i,t|t-1}^{LF}} \cdot \varepsilon_{i,t}^{LF}, \quad (2.3) \]

\[ h_{i,t|t-1}^{LF} = \omega_i^{LF} + \alpha_i^{LF} r_{i,t-1}^2 + \beta_i^{LF} h_{i,t-1}^{LF}, \quad (2.4) \]

with restrictions for non-negativity and stationarity of the variance: \( \omega_i^{LF} > 0 \), \( (\alpha_i^{LF}, \beta_i^{LF}) \geq 0 \), and \( (\alpha_i^{LF} + \beta_i^{LF}) < 1 \). The superscript \( LF \) denotes estimated quantities according to this low frequency model.

If past return information is available at high frequency, the realized variance is the proxy for the past latent variance. As in Andersen et al. (2001b),
the daily realized variance is defined as the cumulative sum of squared \( j \)-th intra-day returns \( r_{t,j} \) over the day \( t \):

\[
RV_t = \sum_{j=1}^{m} r_{t,j}^2
\]  

(2.5)

where \( m \) denotes the number of return observation for each day \( t \). The high frequency GARCH model is as follows:

\[
\begin{align*}
\epsilon_{i,t}^\text{HF} &= h_{i,t|t-1}^\text{HF} \cdot \epsilon_{i,t}, \\
h_{i,t|t-1}^\text{HF} &= \omega_i^\text{HF} + \alpha_i^\text{HF} RV_{i,t-1} + \beta_i^\text{HF} h_{i,t-1}^\text{HF},
\end{align*}
\]

(2.7)

with constraints \( \omega_i^\text{HF} > 0 \), \( (\alpha_i^\text{HF}, \beta_i^\text{HF}) \geq 0 \), and \( (\alpha_i^\text{HF} + \beta_i^\text{HF}) < 1 \), with \( i = 1, \ldots, k \).

A model that uses both the low frequency squared return and the high frequency realized measure is denoted as mixed frequency model and it has the following GARCHX structure:

\[
\begin{align*}
\epsilon_{i,t}^\text{MF} &= h_{i,t|t-1}^\text{MF} \cdot \epsilon_{i,t}, \\
h_{i,t|t-1}^\text{MF} &= \omega_i^\text{MF} + \alpha_i^\text{MF} r_{i,t-1}^2 + \beta_i^\text{MF} h_{i,t-1}^\text{MF} + \gamma_i^\text{MF} RV_{i,t-1},
\end{align*}
\]

(2.9)

with constraints \( \omega_i^\text{MF} > 0 \), \( (\omega_i^\text{MF}, \alpha_i^\text{MF}, \beta_i^\text{MF}, \gamma_i^\text{MF}) \geq 0 \), and

\( (\alpha_i^\text{MF} + \beta_i^\text{MF} + \gamma_i^\text{MF}) < 1 \). This mixed frequency model for volatility corresponds substantially to the HEAVY model of Shephard and Sheppard (2010), except that it does not specify the dynamics of the realized variance process. For multi-step ahead forecasts, the expectation of \( RV_{i,t+h} \) with \( h > 1 \), is approximated with the high frequency forecast of the variance \( h_{i,t+h}^\text{HF} \) given by equation (2.7). The motivation for this is that realized variance has an interpretation as conditional variance. If we assume the true variance is the realized variance this approximation appears rather natural.

All models for the latent conditional volatility are estimated with quasi-
maximum likelihood under the assumption \( \varepsilon_{i,t}^{(\cdot)} \sim N\left(0, h_{i,t}^{(\cdot)}\right) \). The log-likelihood function is

\[
LL_{1}^{(\cdot)} = -\frac{T}{2} \log(2\pi) - \frac{1}{2} \sum_{t=1}^{T} \log \left( h_{t}^{(\cdot)} \right) - \frac{1}{2} \sum_{t=1}^{T} \frac{r_t^2}{h_t^{(\cdot)}},
\]

with the superscript \( (\cdot) \) denoting any of the models for the volatility.

### 2.2.2 Conditional Correlation and Covariance

As for the conditional volatility, the conditional correlation is also differentiated by the frequency of the return information used. All the correlation models follow the DCC-type structure of Engle (2002). In order to cope with non-stationary issues of the correlation driving process present in the original DCC parametrization, I use the corrected DCC version of Aielli (2011) as the low frequency correlation model. The conditional correlation process is modeled as

\[
R_{t|t-1}^{LF} = Q_{t|t-1}^{-\frac{1}{2}} Q_{t|t-1}^{LF} Q_{t}^{-\frac{1}{2}},
\]

where \( Q_{t}^{-\frac{1}{2}} \equiv \text{diag}\left(q_{11,t}^{-\frac{1}{2}}, \ldots, q_{kk,t}^{-\frac{1}{2}}\right) \) is as \((k \times k)\) diagonal matrix with diagonal elements \( q_{i,t}^{-\frac{1}{2}} \) on the main diagonal and zero elsewhere. \( Q_t \) is the correlation driving process, which is specified by Aielli (2011) as

\[
Q_{t|t-1} = \left(1 - a^{LF} - b^{LF}\right) \cdot \overline{Q} + a^{LF} \cdot \left( Q_{t-1}^{-\frac{1}{2}} \varepsilon_{t-1}^{LF} \varepsilon_{t-1}^{LF'} Q_{t-1}^{-\frac{1}{2}}\right) + b^{LF} \cdot Q_{t-1}
\]

with \( \varepsilon_{t}^{LF} = \left[\varepsilon_{1,t}^{LF}, \ldots, \varepsilon_{k,t}^{LF}\right] \) and

\[
\overline{Q} = T^{-1} \sum_{t=1}^{T} \hat{Q}_{t}^{1} \varepsilon_{t}^{LF} \varepsilon_{t}^{LF'} \tilde{Q}_{t}^{-\frac{1}{2}}
\]
2.2. Multivariate Volatility Models

representing the sample estimate of the unconditional correlation matrix. By defining \( \varepsilon_t^* \equiv \tilde{Q}_t^{1/2} \varepsilon_t^{LF} \) it holds that

\[
\begin{align*}
E_{t-1} [\varepsilon_t^*] &= 0, \quad (2.14) \\
E_{t-1} [\varepsilon_t^* \varepsilon_t^{*\prime}] &= Q_t. \quad (2.15)
\end{align*}
\]

The following initial recursions are required to estimate \( \tilde{Q}_t^{1/2} \equiv \text{diag}\left( \tilde{q}_{11,t}, \ldots, \tilde{q}_{kk,t} \right) \):

\[
\tilde{q}_{i,t} = \left( 1 - a^{LF} - b^{LF} \right) + \left( a^{LF} \cdot \varepsilon_{i,t-1}^{LF} + b^{LF} \right) \cdot \tilde{q}_{i,t-1}. \quad (2.16)
\]

Given the presence of the parameters \((a, b)\), the estimation of the unconditional correlation matrix \( \tilde{Q} \) is interconnected to the estimation of the conditional correlation driving process \( Q_t \). This parametrization has the advantage that \( \tilde{Q} = E_t [Q_t] \) and, moreover, the redefined innovation \( \varepsilon_t^* \) is covariance stationary given conditions (2.14) and (2.15).

The high frequency model for the conditional correlation has a similar structure as this low frequency model and the time-varying correlation model of Tse and Tsui (2002). It departs from them for the fact that the realized correlation is used as the driving force. The underlying assumption for the high frequency model is that past conditional correlations are observable as the realized correlation. The asymptotic theory on realized covariance is studied by Barndorff-Nielsen and Shephard (2004a). The realized covariance is defined in this paper as

\[
RCov_t = \sum_{j=1}^{m} S_{t,j} \cdot S_{t,j}', \quad (2.17)
\]

where \( S_{t,j} \) is the vector of \( j \)-th intra-day returns for the day \( t \). From this matrix, the realized correlation matrix is obtained by standardization of each variance-covariance element. For \( i = 1, \ldots, k \) and \( s = 1, \ldots, k \), the realized
correlation between assets $i$ and $j$ at time $t$ is given by

$$R_{Cor_{i,s}}(t) = \frac{RCov_{i,s}(t)}{\sqrt{RV_i \cdot RV_s}}. \quad (2.18)$$

In absence of market microstructure noise, the realized covariance of Barndorff-Nielsen and Shephard (2004a) is a consistent estimator of the integrated covariance of a multivariate continuous semimartingale as the sampling frequency increases $(m \rightarrow \infty)$.

The conditional correlation structure for the high frequency model is the following:

$$R_{HF_{t|t-1}} = \left(1 - a^{HF} - b^{HF}\right) \cdot \overline{RCor} + a^{HF} \cdot R_{Cor_{t-1}} + b^{HF} \cdot R_{HF_{t-1}}, \quad (2.19)$$

where $\overline{RCor}$ is the long run realized correlation obtained by element-wise average of realized correlation matrices over the estimation sample period and it represents the target correlation.

The mixed frequency model for the covariance is inspired by recent works of in the univariate framework by Engle and Gallo (2006), Shephard and Sheppard (2010) and Hansen et al. (2010). However, it does not generalize those models and it also differs from the multivariate HEAVY model of Noureldin et al. (2011) as it does not use the BEKK-type parametrization. The idea is to include both high and low frequency return information in forecasting the covariance matrix. The dynamic correlation structure uses the Aielli (2011) parametrization and it is specified as follows:

$$Q^*_{t|t-1} = \left(1 - a^{MF} - b^{MF}\right) \cdot Q^* + a^{MF} \cdot \left(Q^*_{t-1} \epsilon^M_{t-1} \epsilon^M_{t-1} Q^*_{t-1}^{\epsilon-2}ight) + b^{MF} \cdot Q^*_{t-1} \quad (2.20)$$

and

$$R^{LF*}_{t|t-1} = \text{diag} \left(q^{*-1/2}_{1,t|t-1}, \ldots, q^{*-1/2}_{k,t|t-1}\right) \cdot Q^*_{t|t-1} \cdot \text{diag} \left(q^{*-1/2}_{1,t|t-1}, \ldots, q^{*-1/2}_{k,t|t-1}\right), \quad (2.21)$$

and

$$R^{MF}_{t|t-1} = \delta^{MF} \cdot R^{LF*}_{t|t-1} + \left(1 - \delta^{MF}\right) \cdot R_{Cor_{t-1}}. \quad (2.22)$$

The matrix $R^{LF*}_{t|t-1}$ represents the low frequency correlation matrix. However, it differs from $R^{LF}_{t|t-1}$ in equation (2.11) as it takes residuals $\epsilon^M_{t-1}$, which are returns standardized by mixed frequency volatility estimates. Also, the un-
conditional correlation matrix $\overline{Q}^*$ is obtained similarly as in equations (2.13) and (2.16) but with standardized errors $\varepsilon^{MF}_{t-1}$. The parameter $\delta^{MF}$ represents the weighting for the low frequency correlation matrix.

The covariance matrix for each of the models $(\cdot) \equiv (LF, MF, HF)$ is computed as

$$\Sigma^{(\cdot)}_{t|t-1} = D^{(\cdot)}_{t|t-1} R^{(\cdot)}_{t|t-1} D^{(\cdot)}_{t|t-1}$$ (2.23)

where $D^{(\cdot)}_{t|t-1} \equiv \text{diag} \left[ h^{(\cdot)}_{1,t|t-1}^{1/2}, \ldots, h^{(\cdot)}_{k,t|t-1}^{1/2} \right]$.

Finally, other types of mixed frequency model result as a combination of models for volatility and correlation. Those mixed frequency models are conceived similarly as the mixed frequency model of Halbleib and Voev (2011). I consider the following two other mixed frequency models:

$$\Sigma^{LF-HF}_{t|t-1} = D^{LF}_{t|t-1} R^{HF}_{t|t-1} D^{LF}_{t|t-1}$$ (2.24)

and

$$\Sigma^{HF-LF}_{t|t-1} = D^{HF}_{t|t-1} R^{LF^*+}_{t|t-1} D^{HF}_{t|t-1}$$ (2.25)

where

$$R^{LF^*+}_{t|t-1} = \text{diag} \left( q^{**-1/2}_{1,t|t-1}, \ldots, q^{**-1/2}_{k,t|t-1} \right) \cdot Q^{**}_{t|t-1} \cdot \text{diag} \left( q^{**-1/2}_{1,t|t-1}, \ldots, q^{**-1/2}_{k,t|t-1} \right),$$ (2.26)

$$Q^{**}_{t|t-1} = (1 - a^{**} - b^{**}) \cdot \overline{Q}^{**} + a^{**} \cdot \left( Q^{**}_{t-1} \varepsilon^{HF}_{t-1} \varepsilon^{HF'}_{t-1} Q^{**}_{t-1} \right)^{1/2} + b^{**} \cdot Q^{**}_{t-1}$$ (2.27)

The superscript $LF^{**}$ denotes the low frequency model for the conditional correlation that however takes returns standardized by the high frequency conditional volatility estimate. Those two models are called respectively $LF-HF$ and $HF-LF$ as the former consists of low frequency information based estimates of the volatility and high frequency information based estimates of the correlation and, vice-versa, the latter consists of high frequency information based estimates of the volatility and low frequency information based estimates of the correlation.
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Model Forecast and Data Frequency

The model estimation is carried out with the quasi-maximum likelihood approach by assuming $S_t \mid \mathcal{F}_{t-1} \sim N\left(0, \Sigma_t^{(i)}\right)$, where $S_t$ is the vector of returns ($S_t = [r_1, \ldots, r_k]$) and $\Sigma_t^{(i)}$ is any of the conditional covariance described above. The log-likelihood function is

$$LL_2^{(i)} = -\frac{T_k}{2} \log (2\pi) - \frac{1}{2} \sum_{i=1}^{k} \sum_{t=1}^{T} \log \left|h_{i,t}^{(i)}\right| - \frac{1}{2} \sum_{t=1}^{T} \log \left|R_t^{(i)}\right| - \frac{1}{2} \sum_{t=1}^{T} \epsilon_t^{(i)} R_t^{(i)} - \frac{1}{2} \epsilon_t^{(i)}.'$$

(2.28)

2.3 Covariance Forecasting

2.3.1 Multiperiod Forecast Evaluation Methodology

To assess the forecasting performance of the proposed models an out-of-sample iterative forecasting procedure is applied. The proxy for the true ex-post covariance matrix is the realized covariance. Denoting with $h$ the forecasting horizon, the models generates the following forecasts of the covariance matrix:

$$\Sigma_{t+h|t}^{LF} = D_{t+h|t}^{LF} \cdot R_{t+h|t}^{LF} \cdot D_{t+h|t}^{LF},$$

(2.29)

$$\Sigma_{t+h|t}^{MF} = D_{t+h|t}^{MF} \cdot R_{t+h|t}^{MF} \cdot D_{t+h|t}^{MF},$$

(2.30)

$$\Sigma_{t+h|t}^{HF} = D_{t+h|t}^{HF} \cdot R_{t+h|t}^{HF} \cdot D_{t+h|t}^{HF},$$

(2.31)

$$\Sigma_{t+h|t}^{LF-HF} = D_{t+h|t}^{LF} \cdot R_{t+h|t}^{HF} \cdot D_{t+h|t}^{LF},$$

(2.32)

$$\Sigma_{t+h|t}^{HF-LF} = D_{t+h|t}^{HF} \cdot R_{t+h|t}^{LF} \cdot D_{t+h|t}^{HF}.$$

(2.33)

The forecasting horizons considered goes from 1 to 22. Unless otherwise stated, $h$ denotes the day so that $\Sigma_{t+h|t}$ denotes the forecast for the daily covariance at time $t + h$.

Out-of-sample forecasts for first period ($h = 1$) is obtained by holding the estimated model parameters and iterating estimated volatilities and cor-
relations to the next period. Forecasts for the subsequent periods \((h > 1)\) are obtained by iterating the first period out-of-sample forecast of volatilities and correlations to the next periods and so on. For multi-step ahead forecasts a further complication arises as forecasts for return and realized measures are not available at time \(t + h - 1\), with \(h > 1\). The following approximations are used in my analysis:

\[
\begin{align*}
    r_{i,t+h-1|t}^2 & \quad \Rightarrow h_{i,t+h-1|t}^{LF} \\
    RV_{i,t+h-1|t} & \quad \Rightarrow h_{i,t+h-1|t}^{HF} \\
    \left(\varepsilon_{t+h-1|t}^{*}\right)^\prime \left(\varepsilon_{t+h-1|t}^{*}\right) & \quad \Rightarrow Q_{t+h-1|t} \\
    \left(\varepsilon_{t+h-1|t}^{MF}\right)^\prime \left(\varepsilon_{t+h-1|t}^{MF}\right) & \quad \Rightarrow Q_{t+h-1|t}^* \\
    RC_{t+h-1|t} & \quad \Rightarrow R_{t+h-1|t}^{HF}
\end{align*}
\]

where \(\varepsilon_{t}^{MF} \equiv Q_{t}^{1/2} \varepsilon_{t}^{MF}\) and all other quantities are previously defined. Those approximations seem to be the most natural. Equations (2.34) is commonly used within the (low-frequency) volatility modeling literature. Equations (2.36)-(2.37) find a rationale given the stationarity condition (2.15). The underlying rationale for the approximations (2.35) and (2.38) is that volatility is a unique concept and therefore there is no need to define a separate model for \(RV_t\) and \(RC_{t}\). Those two quantities have the interpretation as conditional variance and conditional correlation and as such their expectations are equivalent to \(h_{t}^{HF}\) and \(R_{t}^{HF}\), respectively.

The evaluation of the forecasting performance is conducted recursively with rolling windows. The in-sample estimation window is fixed to 1000 observations, corresponding to about four years. Out-of-sample forecasts are subsequently generated based on each estimation window and evaluated according to two loss functions: the multivariate mean square error and the multivariate QLIKE loss. The two loss functions have been proposed by Patton and Sheppard (2009) and Laurent et al. (2009) as robust functions to yield model rankings given the approximation error of the unobservable co-
variance matrix. Specifically, they have the following forms, respectively:

\[
L_{t+h|F_t}^{MSE} = \text{tr} \left[ \left( \Sigma_{real,t+h} - \Sigma_{pred,t+h|F_t} \right) \cdot \left( \Sigma_{real,t+h} - \Sigma_{pred,t+h|F_t} \right)' \right], \quad (2.39)
\]

\[
L_{t+h|F_t}^{QLIKE} = \text{tr} \left[ \left( \Sigma_{pred,t+h|F_t}^{-1} \cdot \Sigma_{real,t+h} \right) \cdot \log \left[ \left( \Sigma_{pred,t+h|F_t}^{-1} \cdot \Sigma_{real,t+h} \right) \right] \right] - k, \quad (2.40)
\]

with \( \Sigma_{real,t+h} \) representing the ex-post realized covariance matrix for day \( t+h \) and \( \Sigma_{pred,t+h|F_t} \) representing the predicted covariance matrix with any one of the models from (2.29) to (2.33) with information set at time \( t \). The two loss functions used are also known as Frobenius distance and Stein distance (see Laurent et al., 2009) and they represent generalized versions of the univariate mean square error and univariate QLIKE function described in Patton (2011). The empirical analysis considers ex-post realized covariance with intra-day return data sampled at different frequency, from five minutes to two hours.

2.3.2 Data

The empirical analysis uses high frequency spot exchange rates between four major currency pairs: CHF/USD, EUR/USD, GBP/USD, JPY/USD. The whole sample period ranges from December 2, 1991 to February 29, 2008. The series EUR/USD is a synthetic series constructed by splicing the DEM (German Mark, from December 1991 to December 1998) with the EUR data (from January 1999 to February 2008). The exchange rates are calculated from tick-by-tick Reuters mid quotes, that are averages between the last ask and bid quotes over a 5-minute interval.

The calculation of the realized covariance requires observations to be synchronized and equally spaced among time series. To achieve this, the data is organized in 5-minutes rates, with 288 observation each day. By doing so, the impact of microstructure noises coming from bid-ask quotes is also alleviated (see Danielsson and Payne, 2002). Simple FX returns are then
calculated from five minutes rates. The dataset is further organized according Andersen et al. (2001c). Specifically, a day has been redefined as starting from 21:05GMT of the previous calendar day to 21:00GMT and several days representing week-ends and holidays has been removed. This is motivated by the “ebb and flow” pattern for exchange rates documented in Bollerslev and Domowitz (1993). A list of days excluded from the dataset is provided in the appendix of this chapter.

The number observations in the dataset is as follows. By removing holidays and weekends, $288 \cdot 4056$ five-minutes spaced observations remains, where 4056 is the total number of days left. The first estimation window starts on December 2, 1991 and ends on November 28, 1995 (1000 daily observations). The 1-step-ahead out-of-sample prediction based on this estimation window corresponds to November 29, 1995. A maximum of $h = 22$ forecast horizon is fixed, so that the last estimation window starts on February 3, 2004 and ends January 30, 2008, with the 1-step-ahead forecast, based on this window, corresponding to the subsequent working day January 31, 2008 and the 22-step-ahead forecast corresponding to February 29, 2008. This procedure results in 3035 estimation windows.

Figure 2.1 represents realized variance series for the four pairs of currency returns denominated in US dollar. The sample period includes periods of relative appreciation of the USD (from 1992 to 2002) and relative depreciation (from 2002 to 2008) with respect to the other currencies. It also includes periods of currency crisis and those are the following. The 1992 European Monetary System crisis resulted in a high level of volatility and covariance among European currencies around “Black Wednesday” September 16, 1992. The Latin America currency crisis of 1995 seems to have affected the volatility of all four currency pairs under examination. Instead, the effect of the Asian crisis of 1997-1998 seems to be more contained

---

1With a total number of sample day of 4056, and considering a lead of 22 day for the out-of-sample forecast, I obtain 4034 daily data points for the in-sample estimation (4057-22). By taking an estimation window size of 1000, it is possible to construct 3035 windows on those 4034 days.
as it resulted mainly in a high level of volatility for returns on JPY. As the base currency is the same, the four series display a relatively high level of co-movement and this is especially true for rates involving European currencies.

Figure 2.1: Realized Variance of Exchange Rate Return

NOTE: The figure represents the realized variance of returns on the four currency pairs. Simple returns are used and they represents the unit dollar return of one dollar invested on the foreign currency. The shaded area correspond to the first estimation window.
2.3.3 Empirical Results

I briefly discuss the in-sample analysis while detailed results are omitted as the focus of this chapter is in forecasting. For univariate volatility models the estimated parameters are similar among the different models, despite the different proxy used for the latent conditional volatility. This means that the volatility persistence is captured in a similar way, whatever is the volatility proxy used. Figure 2.2 represents the rolling window coefficient estimates. The estimated high frequency parameter $\alpha^{HF}$ moves closely with the low frequency parameter $\alpha^{LF}$ and the mixed frequency parameters $\alpha^{MF} + \gamma^{MF}$ for all four series. Also, the estimated $\beta(\cdot)$ are similar among the three models. The estimated value for $\alpha(\cdot)$ ranges between 0.01 and 0.17, while values for $\beta(\cdot)$ range between 0.6 and 0.99.

Concerning conditional correlations, the estimated parameters differs among the models given the different proxy for past correlations. Figure 2.3 represents rolling window coefficient estimates for the conditional correlation. Mean values for $a^{LF}$ and $b^{LF}$ are 0.02 and 0.95, while mean values for $a^{HF}$ and $b^{HF}$ are 0.18 and 0.81. The coefficient $\delta^{MF}$, denoting the weight associated to the low frequency proxy, in the mixed frequency model, has a mean of 0.99 and is lower in the second part of the sample which is overall a calm period. This indicates that high frequency correlation information may be especially useful to model dynamics in calm periods.

The out-of-sample analysis takes into account the fact that the realized covariance is still a noisy proxy for the true covariance. For model estimation, all realized quantities required by the high and mixed frequency information models are computed from 5 minutes intra-day returns, which is a popular and naive high sampling frequency. However, given the presence of microstructure noises, the realized covariance from 5 minutes return data may still be an imperfect approximation of the true covariance. De Pooter
et al. (2008) even argue that for portfolio applications the optimal sampling frequency should be between 30 and 65 minutes. I study the accuracy of covariance forecasts when the true ex-post covariance is the realized covariance with a sampling frequency that varies between 5 minutes and 120 minutes.

The following point emerges from the out-of-sample analysis. When the true covariance matrix is the realized covariance computed from 5 minute returns, a model that uses high frequency information for the correlation seems to perform better. Instead, when other sampling frequencies are used for the ex-post realized covariance, a low frequency or a mixed frequency model for correlation tend to perform better. Concerning the first stage volatility estimation, it seems that high return information does not add a value. Table 2.1 summarizes the losses resulting from each model forecast. Loss information are reported in mean for the 16 elements of the covariance matrix.

The forecast performance deteriorates as the forecasting horizon increases. However the ranking among the models for different forecasting horizons, based on both \( \text{MSE} \) and \( \text{QLIKE} \) loss functions, is maintained. Overall, the \( \text{MSE} \) appears to favor the low frequency and mix frequency models, while the \( \text{QLIKE} \) loss function seems to favor the \( \text{LF} - \text{HF} \) model. This loss function is based on the ratio between realized and predicted covariance matrix and therefore it introduce some asymmetries in the penalty inflicted between over prediction and under prediction.

To ascertain the predictive accuracy of the models for realized covariance, the model confidence set approach of Hansen et al. (2011) is applied. Based on the two loss functions the model confidence set methodology tests recursively whether the model forecasts are significantly different and concurrently exclude models with significantly lower performance than the other models in the set. The interpretation for the surviving models in the confi-
2.4. Out-of-Sample Portfolio Realized Volatility

The confidence set is that they have equal predictive ability and they yield a better forecast than the other models, given a confidence level. I apply the following "range" test statistics discriminate the models:

\[
 t_R = \max_{p,q \in M_{\text{set}}} \frac{|\bar{d}_{p,q}|}{\sqrt{\text{var} (\bar{d}_{p,q})}},
\]  

(2.41)

where \( \bar{d}_{p,q} \) is the mean loss differential between each pair combination of models \( p \) and \( q \). The significance level of the test is fixed at \( \alpha = 0.15 \) and \( \alpha = 0.25 \), with the test based on the latter significance level being more stringent. The included models in \( M_{15\%} \) and \( M_{25\%} \) are however quite similar. Table 2.2 summarizes the models selected in the final confidence set.

The superiority of one model over another is not very neat. Results based on the MSE and QLIKE tend to be different, moreover the model ranking given the imperfect proxy for the ex-post realized covariance is not always maintained. Specifically, for the MSE, a difference in ranking emerges between an ex-post realized covariance that uses 5 minutes sampling and 10 minutes sampling, and for the QLIKE loss a difference in ranking emerges between 30 minutes sampling and 60 minutes sampling. Those results rise some question about the adequacy of the two loss functions when comparing covariance forecasts. Overall, the forecasting model with low frequency volatility and high frequency correlation (\( LF - HF \)) is preferred.

2.4 Out-of-Sample Portfolio Realized Volatility

2.4.1 Investment Strategy

The portfolio evaluation is proposed as a mean to measure the economic value of model forecasts. In particular, given that some models use low
frequency return information and others use high frequency return information or combinations thereof, it serves also as a mean to characterize the economic content of using high frequency realized measures in portfolio selection. The perspective adopted in constructing portfolios is that of an investor who is constrained to take positions on foreign currencies. This investor may be represented by fund manager who takes long positions only on foreign currencies. Example of funds with this profile are the exchange traded funds that offer hedge against depreciation of the home currency, in this case the USD. Another type of investor who is interested in investing in foreign currencies is a speculator who takes both long and short positions.

The investment strategies adopted are volatility timing strategies. Accordingly, the optimal portfolio allocation is determined by the forecast of the conditional covariance matrix. In deciding the optimal weights for portfolio allocation, the minimum variance portfolio is considered the optimal portfolio. This portfolio is on the Markowitz (1952) efficient frontier and its optimal weights do not depend on the mean forecast. For investments without short selling constraints, the following analytical solution for the optimal weights exist:

\[ w^*_{t+h} = \frac{\left( \Sigma_{t+h|F_t}^{predicted} \right)^{-1} \cdot 1}{1' \cdot \left( \Sigma_{t+h|F_t}^{predicted} \right)^{-1} \cdot 1}, \] (2.42)

where \( 1 \) is a the \((k \times 1)\) vector of ones and \( \Sigma_{t+h|F_t}^{predicted} \) is defined as the average of predicted covariances between time \( t + 1 \) and time \( t + h \). Different optimal weights are calculated depending on each model for the covariance. Therefore, the volatility timing strategy yields a different performance with respect to each volatility model analyzed.

A dynamic trading strategy is considered. The portfolio of FX currencies

\[ w^*_{t+h} = \frac{\left( \Sigma_{t+h|F_t}^{predicted} \right)^{-1} \cdot 1}{1' \cdot \left( \Sigma_{t+h|F_t}^{predicted} \right)^{-1} \cdot 1}, \] (2.42)
2.4. Out-of-Sample Portfolio Realized Volatility

is rebalanced within fixed intervals. The analysis focuses on rebalancing every trading day, and every 5 and 30 trading days. By doing so I exploit forecasts of the covariance matrix that are available for the one horizon ahead and for multiple horizons ahead. The period of investment is matched accordingly for the different rebalancing strategies. For instance, the first portfolio formation is in date November 28, 1995. Thus, the portfolio weights are optimized according to the forecast of the covariance matrix for November 29, 1995 in case of daily rebalancing and according to the forecast of the average of the covariance matrices from November 29, 1995 to January 3, 1996 (22 business days) for rebalancing every 22 days.

The investment performance is measured according to two different indicators. First, I consider the ex-post average daily realized volatility of currency minimum variance portfolios

\[
\sqrt{RV_{t+h}^{portfolio}} = \sqrt{w_{\ast}^{t+h} - \Sigma_{realized}^{t+h} w_{\ast}^{t+h}}. \tag{2.43}
\]

This represents a direct measure of the forecast accuracy as optimal asset weights based on better predictions of the realized covariance more readily achieve the goal of minimizing the ex-post portfolio realized volatility. Second, I consider the portfolio turnover for each volatility timing investment strategy. This is an indicator for transaction costs and it is measured as

\[
Turnover_{t+h} = \sum_{i=1}^{k} |w_{i,t+h-}^{\ast} - w_{i,t}^{\ast}|, \tag{2.44}
\]

where \(w_{i,t}^{\ast}\) is the ex-ante optimal weight associated to the asset \(i\) at time \(t\). When the absolute change in portfolio weights is relatively higher, the investment strategy dictates a higher trading volume during the portfolio rebalancing days, hence higher transaction costs. As a benchmark for the volatility timing strategy, I consider a naive strategy that invests in equally weighted foreign currencies and I call the portfolio formed from this strategy the “1/n” portfolio for which the change in weights is zero.
Chapter 2. Multivariate Exchange Rate Volatility: Model Forecast and Data Frequency

2.4.2 Portfolio Evaluation Results

Results for the performance in terms of the realized volatility and the turnover of FX portfolios are reported in Table 2.3. On average, the volatility timing strategy based on all models achieves the goal of decreasing the portfolio realized variance. This is especially true if short selling is not allowed. The mean realized volatility of the naive “1/n portfolio” is generally higher than the one of portfolios formed according to volatility timing strategies. This result is assuring as, indeed, volatility timing strategies have a value added.

Realized volatilities obtained when short selling is allowed, compared with the ones obtained when short selling is not allowed, are generally higher. However, some exceptions are present for volatility timing with high frequency forecasts of correlation. Consistently with the previous literature, the short selling constraint works out as a kind of “insanity filter” (see Patton, 2004), which is more pronounced when forecasts are less precise. Forecasts obtained with the high frequency model for the correlation, however, tend to be more accurate and therefore short-selling does not increase the portfolio realized volatility.

An investor may expect the existence of a trade-off between the portfolio realized volatility and transaction costs. To successfully minimize portfolio volatility, transaction costs should be relatively higher on the long run. This reveals not to be true from the analysis. Volatility timing with models which use high frequency information for the correlation tends to yield also lower transaction costs, especially when portfolio rebalancing is every several days. For the daily rebalancing differences in transaction cost are relatively small. The volatility timing strategies require an average daily rebalancing of the portfolio assets between 5.5% and 6.1% if short selling is not allowed and between 6.7% and 11.6% if short selling is allowed. When rebalancing occurs every several days the turnover increases substantially and, with monthly rebalancing, it ranges between 18.4% ($LF - HF$ model).
to 21.6% (HF – LF model) when short selling is not allowed, and between 21% (HF model) to 39.5% (LF model) when short selling is allowed.

Overall, the volatility timing investing analysis confirms results from the previous section. The high frequency forecasting model for the correlation is the best choice yielding a good compromise between the ex-post portfolio realized volatility and the turnover rate.

2.5 Model Backtesting

2.5.1 Density Forecast

From a general forecasting perspective, an important aspect of the multivariate volatility models is that they are able to produce adequate density forecasts. In this section I study whether forecast errors follow the standard normal law as the models assume. I employ the probability integral transform technique proposed by Diebold et al. (1998) to assess qualitatively the adequacy of the data generating process and the Berkowitz (2001) backtesting procedure to ascertain the confidence on the data generating assumption. The quantity of interest is the density of standardized conditional returns.

Realized return innovations, given the forecast of the conditional variance, are mapped to their probability integral transform (i.e., cumulative density values). The return process for each of the volatility models reads as

\[ r_t = \sqrt{h_{t|t-1}} \cdot \varepsilon_t, \quad \varepsilon_t \sim \text{i.i.d. } N(0,1), \]  

with \( h_{t|t-1} \) being the predicted quantity out-of-sample. The transformed series of \( \varepsilon_t \) is denoted as

\[ x_t = \int_{-\infty}^{\varepsilon_t} f_t(u) \, du \]  

where \( f(\cdot) \) is the probability density function, in this case a standard normal
When the forecast innovations are indeed normal in law, then \( x_t \sim \text{i.i.d.} \ U(0, 1) \).

The uniform transformation is applied to innovations of each univariate FX return series, in order to evaluate the adequacy of the univariate \textit{GARCH}-type models. Moreover, to evaluate the adequacy of the multivariate \textit{DCC}-type specification, the probability integral transform is also applied to innovations of FX portfolios, which use predicted covariance information. I consider the equally weighted portfolio and minimum variance portfolios of the previous session, with and without short sale constraints, serving as a robustness check. For each of the four FX return series, I show in Figures 2.4-2.7 the empirical probability density function of innovations \( \varepsilon_t \), the histogram of \( x_t \), and the correlogram of \( x_t \). Results for portfolios are represented in Figures 2.8-2.10.

All the series of standardized returns appear to be approximately standard normal and the histogram for the transformed series is approximately a rectangle. Only one difference emerges between the different density forecasts. The unconditional variance of the portfolio return residual’s distribution is higher when the covariance forecast model used takes high frequency information for the conditional correlation. This is visible by comparing the
empirical PDF of rows 3 and 4 to the other rows in Figures 2.8-2.10. Moreover, it is worth to mention that the transformed series $x_t$ of the equally weighted portfolio (see fig. 2.8), and of the CHF and EUR return series (see fig. 2.4 and 2.5), exhibit a small first order autocorrelation, which violates the independence assumption.

To quantify the confidence on the normality assumption, I apply the Berkowitz (2001) likelihood ratio test on innovations $e_t$\(^3\). Under the null, the series has zero mean, unit variance, and it is not autocorrelated across observations. Using a first order $AR(1)$ process,

$$e_t = \mu + \rho e_{t-1} + e_t, \quad t = 1, \ldots, T$$

(2.47)

the test statistics is computed as

$$LR_B = -2 \left[ L(0,1,0) - L(\hat{\mu}, \hat{\sigma}^2, \hat{\rho}) \right],$$

(2.48)

where $L(0,1,0)$ is the log-likelihood of the standard normal and

$$L(\mu, \sigma^2, \rho) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log \left( \frac{\sigma^2}{1 - \rho^2} \right) - \frac{[\varepsilon_1 - \mu / (1 - \rho)]^2}{2\sigma^2 / (1 - \rho^2)}$$

$$- \frac{T-1}{2} \log(\sigma^2) - \sum_{t=2}^{T} \left[ \frac{e_t^2}{2\sigma^2} \right]$$

(2.49)

is the exact log-likelihood of the Gaussian $AR(1)$. $\sigma^2$ is the variance of the series $e_t$. Under the null, $LR_B \sim \chi^2_3$.

The results of the Berkowitz (2001) likelihood ratio test are reported in Table 2.4. All the residual series of individual currencies pass the test at about 5% level. The residuals from the GBP are the most close to the i.i.d. standard normal, while residuals from the CHF and EUR series are only weakly significantly i.i.d. standard normal given the presence of the first order autocorrelation. The different $GARCH$-type volatility models produce

\(^3\)Generally, the test works with any class of continuous distribution, by re-transforming the uniform $x_t$ series into a standard normal with the inverse normal function.
similar return density predictions, with the high frequency model producing residuals that are slightly “less” i.i.d. standard normal, by considering the lower p-value of the test. Concerning multivariate models, they can be neatly discriminated based on their ability to produce standard normal portfolio densities. Specifically, covariance models that use high frequency correlation information yield portfolio return innovations which are not normal.

From the previous sections, I highlighted the importance to use high frequency information for the correlation in the perspective of forecasting the realized covariance. However, from the analysis in this section, model that takes high frequency correlation information produce return residuals that deviated from the standard normal law. Overall, the predicted covariance of the high frequency correlation model tends to be lower than that of the low frequency correlation model. Giot and Laurent (2004) find that daily returns standardized by the square root of the 1-day ahead forecast of the daily realized volatility are not normally distributed. I find that it is especially in the multivariate case that FX returns standardized by the predicted high frequency volatility, which are not normally distributed. Notice that, even if the Berkowitz (2001) test applies to a univariate series, portfolio returns represent combinations of individual assets and portfolio volatility represents the generalized volatility, i.e. multivariate volatility.

### 2.5.2 VaR Backtesting

For risk managers, the tail of the distribution is often more relevant than the interior of the distribution and when predictions of large losses are inaccurate then the risk models are deemed to fail. While previously I focused on the entire distribution of returns, the focus here is only on the left tail distribution. I examine the 1-day ahead predicted Value-at-Risk with the different multivariate volatility models in order to assess their accuracy in managing
2.5. Model Backtesting

risk. VaR emerged as the most prominent measure for downside risk and it represents the threshold value of losses over a fixed period of time, in this case one day, with a fixed probability $1 - \alpha$.

A correctly specified VaR model satisfies

$$P (r_{t+1} \leq \text{VaR}_{t+1,\alpha} | \mathcal{F}_t) = \alpha.$$  

(2.50)

To backtest the predicted $\text{VaR}_{t+1}$, I calculate the percentage of return exceedance over the predicted VaR, given return realizations $r_{t+1}$. The return exceedance is called failure rate and it is calculated as

$$\frac{1}{T} \sum_{t=1}^{T} 1\{r_{t+1} \leq \text{VaR}_{t+1,\alpha}\},$$

where $\text{VaR}_{t+1}$ is predicted by using each of the volatility model proposed. Similarly to the previous session, I evaluate first the univariate GARCH-type models by studying the correct risk coverage for each series of FX return and the multivariate DCC-type models by studying the correct risk coverage for portfolios (equally weighted, minimum variance with no short sales, minimum variance with short sales) of FX assets. Given the Gaussian assumption of return innovation, the one day ahead $\text{VaR}_{t+1}$ is predicted as $z_\alpha \sqrt{h_{t+1|t}}$, with $z_\alpha$ being the left $\alpha$ quantile of the standard normal distribution and $h_{t+1|t}$ being the predicted conditional variance for the asset or the portfolio with each of the risk models.

Table 2.5 reports the $\text{VaR}_\alpha$ coverage rate (i.e., return failure rate), with $\alpha = (0.01, 0.05, 0.1)$, and the p-values of the Kupiec (1995) likelihood ratio test of correctly predicted coverage rate. The Kupiec (1995) likelihood ratio test statistics is given by

$$LR_K = -2 \left\{ \log \left[ (1 - \alpha)^{T - n^*} \alpha^{n^*} \right] - \log \left[ \left( 1 - \frac{n^*}{T} \right)^{T - n^*} \left( \frac{n^*}{T} \right)^{n^*} \right] \right\},$$

(2.51)

where $T$ is the total number of observation and $n^* = \sum_{t=1}^{T} 1\{r_{t+1} \leq \text{VaR}_{t+1,\alpha}\}$ is the number of return violations. Under the null hypothesis $\frac{n^*}{T} = \alpha$, the likelihood ratio test statistics is $LR_K \sim \chi^2_1$. 
The percentage of violation is close to the theoretical one for all the individual currency returns. Only for the JPY series the $\alpha = 0.05$ and $\alpha = 0.1$ quantiles are over-predicted. Except this, the tail risk of all other series is adequately predicted with the univariate GARCH-type models. Therefore the conditional univariate normal model assumption appears appropriate for all individual return series. Concerning portfolio of currency returns, covariance models that use high frequency correlation information do not predict or only marginally predict correctly the tail risk. This confirms results from previous analysis. The use of high frequency information for the correlation tend to under-predict the tail risk.

To give an intuitive representation of VaR predictions, Figure 2.11 reports forecasts of the 1-day ahead VaR at $\alpha = 0.01$ for the portfolio of equally weighted currencies. It is noticeable that the covariance models using high frequency information for the correlation (HF and LF − HF) produce VaR forecasts which are considerably lower than forecasts of the other models.

## 2.6 Conclusion

I analyzed the out-of-sample forecasting accuracy of different multivariate volatility models. All the models are based on the statistical decomposition of the conditional covariance matrix in terms of conditional volatilities and correlations. When the realized covariance matrix approximate the true latent covariance matrix, the best forecasting model is the one that uses high frequency information for the correlation.

A volatility timing strategy is implemented in order to evaluate the economic value added of having a more precise forecast of the realized covariance. The ex-post realized volatility of the optimal portfolio is indeed at its
optimal level when forecasts are more precise and thus the volatility timing strategy with the covariance forecast based on the realized correlation is preferred. However, by studying the forecast return distribution, models that use high frequency correlation do not produce standard normal innovations. This implies that returns standardized by the multivariate realized volatility are not standard normal. Overall, to forecast the multivariate realized volatility it is recommended to use past realized correlation information. Forecasts of the $GARCH$ mixed frequency model, of the type of Shephard and Sheppard (2010), do not differ dramatically from standard $GARCH$ models.

An important point that is left over from this paper is the evaluation of the robustness of the two statistical loss functions used when different proxies for the latent volatility are available. This research agenda has been already addressed by Laurent et al. (2009). However, would the information advantage (conditioning information) of some volatility models, in terms of the return data frequency observed, distort the ranking yield by the loss functions? When this is the case, loss functions that are able to impose a penalty on the information advantage may be preferable. This point is left over for future research.
NOTE: The figure represents rolling window estimated parameters for univariate GARCH-type models. On the row dimension of the figure, I report estimates for the different currency series. The plots on the left column of the figure depict the coefficients $\alpha^{LF}$, $\alpha^{HF}$, and $\alpha^{MF}$, while the plots on the right column delineate the autoregressive coefficients $\beta^{LF}$, $\beta^{HF}$, and $\beta^{MF}$. Each estimation window consists of 1000 daily observations.
Figure 2.3: Rolling Window Correlation Parameters

NOTE: The figure represents rolling window estimated parameters for DCC-type models. The first two plots report respectively the coefficients $a^{(\cdot)}$ and $b^{(\cdot)}$ for the low and high frequency models. The bottom plot reports the coefficient $\delta^{MF}$, denoting the estimated weight on the low frequency correlation proxy, within the mixed frequency model. Each estimation window consists of 1000 daily observations.
Chapter 2. Multivariate Exchange Rate Volatility: Model Forecast and Data Frequency

Table 2.1: Average Forecast Losses

<table>
<thead>
<tr>
<th>Forecasting Horizon (Rolling Windows)</th>
<th>Ex-post Realized Covariance Sampling</th>
<th>Multivariate MSE Loss Forecasting Model</th>
<th>Multivariate QLIKE Loss Forecasting Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Day (3035 daily obs.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 mins</td>
<td>1.10</td>
<td>1.10</td>
<td>1.09</td>
</tr>
<tr>
<td>10 mins</td>
<td>1.33</td>
<td>1.33</td>
<td>1.35</td>
</tr>
<tr>
<td>20 mins</td>
<td>1.28</td>
<td>1.28</td>
<td>1.32</td>
</tr>
<tr>
<td>30 mins</td>
<td>1.51</td>
<td>1.51</td>
<td>1.56</td>
</tr>
<tr>
<td>1 hour</td>
<td>1.76</td>
<td>1.76</td>
<td>1.81</td>
</tr>
<tr>
<td>2 hours</td>
<td>1.29</td>
<td>1.29</td>
<td>1.27</td>
</tr>
<tr>
<td>1 Week (3035 daily obs.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 mins</td>
<td>1.57</td>
<td>1.57</td>
<td>1.58</td>
</tr>
<tr>
<td>10 mins</td>
<td>1.50</td>
<td>1.51</td>
<td>1.52</td>
</tr>
<tr>
<td>20 mins</td>
<td>1.49</td>
<td>1.49</td>
<td>1.51</td>
</tr>
<tr>
<td>30 mins</td>
<td>1.73</td>
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<td>1.76</td>
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<td>1 hour</td>
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<td>1.93</td>
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<td>2 hours</td>
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<td>30 mins</td>
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<td>1.60</td>
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<tr>
<td>1 hour</td>
<td>1.55</td>
<td>1.56</td>
<td>1.58</td>
</tr>
<tr>
<td>2 hours</td>
<td>1.80</td>
<td>1.81</td>
<td>1.84</td>
</tr>
<tr>
<td>2 hours</td>
<td>2.01</td>
<td>2.01</td>
<td>2.05</td>
</tr>
</tbody>
</table>

NOTE: The table summarizes the forecasting accuracy of the different models for the conditional covariance matrix according to the multivariate MSE (Frobenius distance) and QLIKE loss (Stein distance) given that the true covariance matrix is the realized covariance, which is analyzed at different sampling frequency. Forecasting horizons of 1, 5, 22 days ahead are reported. Each forecast is based on a rolling estimation window of 1000 observations. Losses are expressed as aggregate mean of the 16 elements of the covariance matrix, over the whole sample period. Returns used to compute the covariance matrix are in percentage.
2.6. Conclusion

Table 2.2: Model Confidence Set

<table>
<thead>
<tr>
<th>Forecasting Horizon</th>
<th>Ex-post Realized Covariance Sampling</th>
<th>Multivariate MSE Loss</th>
<th>Multivariate QLIKE Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5 mins</td>
<td>10 mins</td>
<td>20 mins</td>
</tr>
<tr>
<td>1 Day</td>
<td>++</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td></td>
<td>++</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>1 Week</td>
<td>++</td>
<td>++</td>
<td>++</td>
</tr>
<tr>
<td>1 Month</td>
<td>++</td>
<td>++</td>
<td>++</td>
</tr>
</tbody>
</table>

**NOTE:** The table denotes with “+” (and in a lighter color) the final selected models in the 15% confidence set ($MCS_{15\%}$) and with “++” (and in darker color) the selected models in the 25% confidence set ($MCS_{25\%}$), with $MCS_{25\%} \subseteq MCS_{15\%}$. Both the multivariate $MSE$ (Frobenius distance) and the $QLIKE$ loss (Stein distance) are considered with forecasting horizons of 1, 5 and 22 days ahead. The “range” test statistics is used to discriminate the models. A block bootstrap procedure with average block size of 15 observations and 2000 replications is used to establish equal predictive ability of the surviving models.
Table 2.3: Portfolio Realized Volatility and Turnover

<table>
<thead>
<tr>
<th>Rebalancing</th>
<th>Ex-post Realized Covariance Sampling</th>
<th>Short Selling Not Allowed</th>
<th>Short Selling Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/n Portfolio</td>
<td>Volatility Timing Model</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LF-LF</td>
<td>MF-MF</td>
<td>HF-HF</td>
</tr>
<tr>
<td>1 Day (3034 obs)</td>
<td>5 mins</td>
<td>7.67 7.67 7.62 7.33 7.31 7.65</td>
<td>8.34 8.29 7.34 7.32 8.32</td>
</tr>
<tr>
<td></td>
<td>10 mins</td>
<td>7.54 7.22 7.22 7.06 7.05 7.25</td>
<td>7.67 7.64 7.06 7.05 7.66</td>
</tr>
<tr>
<td></td>
<td>30 mins</td>
<td>7.28 6.76 6.76 6.71 6.70 6.77</td>
<td>6.93 6.91 6.70 6.70 6.93</td>
</tr>
<tr>
<td></td>
<td>2 hours</td>
<td>7.12 <strong>6.42</strong> 6.42 6.45 6.46 6.43</td>
<td>6.43 <strong>6.42</strong> 6.44 6.45 6.42</td>
</tr>
<tr>
<td>Mean Turnover</td>
<td>- 0.059 0.060 0.056 0.061 <strong>0.055</strong></td>
<td></td>
<td>0.114 0.116 <strong>0.067</strong> 0.073 0.108</td>
</tr>
</tbody>
</table>

| 1 week (606 obs) | 5 mins | 7.77 7.74 7.74 7.44 7.42 7.77 | 8.44 8.39 7.44 7.42 8.42 |
|                  | 10 mins| 7.66 7.36 7.36 7.19 7.18 7.38 | 7.79 7.76 7.19 7.18 7.78 |
|                  | 20 mins| 7.52 7.06 7.06 6.97 6.96 7.08 | 7.31 7.29 **6.96** 6.96 7.31 |
|                  | 30 mins| 7.46 6.94 6.94 **6.88** 6.88 6.95 | 7.11 7.09 **6.87** 6.87 7.10 |
|                  | 1 hour | 7.46 6.84 6.83 6.83 6.82 6.85 | 6.91 6.89 **6.81** 6.81 6.91 |
|                  | 2 hours| 7.44 **6.72** 6.72 6.75 6.76 6.73 | 6.72 **6.71** 6.74 6.74 6.72 |
| Mean Turnover | - 0.127 0.127 **0.111** 0.119 0.117 | 0.238 0.237 **0.127** 0.135 0.228 |

| 1 Month (137 obs) | 5 mins | 7.82 7.83 7.83 7.52 7.50 7.87 | 8.49 8.44 **7.53** 7.50 8.48 |
|                  | 10 mins| 7.73 7.45 7.45 7.29 7.27 7.49 | 7.87 7.83 **7.29** 7.27 7.86 |
|                  | 20 mins| 7.58 7.17 7.17 7.08 7.07 7.19 | 7.41 7.38 **7.07** 7.06 7.41 |
|                  | 30 mins| 7.54 7.05 7.05 7.00 6.99 7.07 | 7.20 7.18 **6.99** 6.99 7.21 |
|                  | 1 hour | 7.55 6.97 6.97 **6.96** 6.96 6.98 | 7.03 7.01 **6.95** 6.95 7.03 |
|                  | 2 hours| 7.55 **6.87** 6.87 6.90 6.91 6.88 | 6.86 **6.85** 6.89 6.90 6.87 |
| Mean Turnover | - 0.214 0.215 0.185 **0.184** 0.216 | 0.395 0.386 **0.210** 0.212 0.380 |

NOTE: The table reports the mean realized volatility and turnover of currency portfolios. The currency portfolios are the equally weighted portfolio and the minimum variance portfolios, with and without short selling, optimized according to the different model forecasts of the covariance matrix. The statistics are reported according to portfolio rebalancing every 1, 5, and 22 days. The realized volatility is expressed in percentage and annualized over 252 days.
NOTE: The figure delineates the empirical probability density function of standardized returns (first column) on the CHF currency, the histogram of the uniform probability transformed series $x_t$ (second column), and the correlogram of $x_t$ (third column), obtained with the different forecasting model for volatility (on the row dimension). The empirical probability density function of standardized returns is the black curve, which is contrasted with the theoretical standard normal distribution delineated in red.
Figure 2.5: Probability Integral Transform - Standardized Returns on EUR

NOTE: The figure delineates the empirical probability density function of standardized returns (first column) on the EUR currency, the histogram of the uniform probability transformed series $x_t$ (second column), and the correlogram of $x_t$ (third column), obtained with the different forecasting model for volatility (on the row dimension). The empirical probability density function of standardized returns is the black curve, which is contrasted with the theoretical standard normal distribution delineated in red.
2.6. Conclusion

Figure 2.6: Probability Integral Transform - Standardized Returns on EUR

NOTE: The figure delineates the empirical probability density function of standardized returns (first column) on the GBP currency, the histogram of the uniform probability transformed series $x_t$ (second column), and the correlogram of $x_t$ (third column), obtained with the different forecasting model for volatility (on the row dimension). The empirical probability density function of standardized returns is the black curve, which is contrasted with the theoretical standard normal distribution delineated in red.
Chapter 2. Multivariate Exchange Rate Volatility: Model Forecast and Data Frequency

Figure 2.7: Probability Integral Transform - Standardized Returns on JPY

![Figure 2.7: Probability Integral Transform - Standardized Returns on JPY](image)

NOTE: The figure delineates the empirical probability density function of standardized returns (first column) on the JPY currency, the histogram of the uniform probability transformed series $x_t$ (second column), and the correlogram of $x_t$ (third column), obtained with the different forecasting model for volatility (on the row dimension). The empirical probability density function of standardized returns is the black curve, which is contrasted with the theoretical standard normal distribution delineated in red.
2.6. Conclusion

Figure 2.8: Probability Integral Transform - Equally Weighted Portfolio

NOTE: The figure reports the empirical probability density function of standardized returns (first column), the histogram of the uniform probability transformed series $x_t$ (second column), and the correlogram of $x_t$ (third column), of the equally weighted FX portfolio. Standardized returns are conditional on the different model forecast of the covariance matrix (on the row dimension of the figure). The empirical probability density function of standardized returns is the black curve, which is contrasted with the theoretical standard normal distribution delineated in red.
Figure 2.9: Probability Integral Transform - No Short Selling Min. Var. Portfolio

NOTE: The figure reports the empirical probability density function of standardized returns (first column), the histogram of the uniform probability transformed series $x_t$ (second column), and the correlogram of $x_t$ (third column), of the minimum variance FX portfolio without short selling. Standardized returns are conditional on the different model forecast of the covariance matrix (on the row dimension of the figure). The empirical probability density function of standardized returns is the black curve, which is contrasted with the theoretical standard normal distribution delineated in red.
2.6. Conclusion

Figure 2.10: Probability Integral Transform - Short Selling Min. Var. Portfolio

NOTE: The figure reports the empirical probability density function of standardized returns (first column), the histogram of the uniform probability transformed series $x_t$ (second column), and the correlogram of $x_t$ (third column), of the minimum variance FX portfolio with short selling. Standardized returns are conditional on the different model forecast of the covariance matrix (on the row dimension of the figure). The empirical probability density function of standardized returns is the black curve, which is contrasted with the theoretical standard normal distribution delineated in red.
## Table 2.4: Berkowitz Likelihood Ratio Test

<table>
<thead>
<tr>
<th>Return Series / Portfolio</th>
<th>GARCH Forecasting Model</th>
<th></th>
<th></th>
<th></th>
<th>Covariance Forecasting Model</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LF</td>
<td>MF</td>
<td>HF</td>
<td>LF-LF</td>
<td>MF-MF</td>
<td>HF-HF</td>
<td>LF-HF</td>
<td>HF-LF</td>
</tr>
<tr>
<td>CHF</td>
<td>4.9%</td>
<td>5.0%</td>
<td>4.6%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EUR</td>
<td>5.1%</td>
<td>5.1%</td>
<td>4.8%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GBP</td>
<td>57.2%</td>
<td>56.3%</td>
<td>18.6%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JPY</td>
<td>6.5%</td>
<td>5.9%</td>
<td>6.1%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1/n</td>
<td>6.8%</td>
<td>7.3%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>8.2%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No Short Min. Variance</td>
<td>27.7%</td>
<td>25.5%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>5.7%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Short Min. Variance</td>
<td>15.4%</td>
<td>17.3%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>1.4%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: The table reports p-values of the Berkowitz (2001) likelihood ratio test under the joint null hypothesis that the return residuals are i.i.d. standard normal against the alternative that they follow an AR(1) process with $\mu \neq 0$ and $\sigma \neq 1$. The test is applied to individual currency return series (upper panel) and portfolios of currency returns (lower panel) with the conditional volatility predicted by the different models.
2.6. Conclusion

Table 2.5: VaR Failure Rates

<table>
<thead>
<tr>
<th>Significance Level</th>
<th>Return Series / Portfolio</th>
<th>GARCH Forecasting Model</th>
<th>Covariance Forecasting Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>LF</td>
<td>MF</td>
</tr>
<tr>
<td>CHF</td>
<td>1.09% (0.634)</td>
<td>1.09% (0.634)</td>
<td>1.05% (0.765)</td>
</tr>
<tr>
<td>EUR</td>
<td>1.15% (0.408)</td>
<td>1.19% (0.317)</td>
<td>1.22% (0.241)</td>
</tr>
<tr>
<td>GBP</td>
<td>1.45% (0.02)</td>
<td>1.45% (0.02)</td>
<td>1.48% (0.013)</td>
</tr>
<tr>
<td>VaR 1% Failure Rates</td>
<td>0.89% (0.533)</td>
<td>0.89% (0.533)</td>
<td>0.89% (0.533)</td>
</tr>
<tr>
<td>1/n No Short Min.</td>
<td></td>
<td>0.86% (0.416)</td>
<td>0.86% (0.416)</td>
</tr>
<tr>
<td>Variance Short Min.</td>
<td></td>
<td>0.12% (0.017)</td>
<td>1.15% (0.001)</td>
</tr>
<tr>
<td>Variance</td>
<td></td>
<td>1.25% (0.017)</td>
<td>1.22% (0.017)</td>
</tr>
<tr>
<td>Var 5% Failure Rates</td>
<td></td>
<td>4.74% (0.515)</td>
<td>4.71% (0.462)</td>
</tr>
<tr>
<td>EUR</td>
<td>4.81% (0.63)</td>
<td>4.78% (0.571)</td>
<td>4.74% (0.515)</td>
</tr>
<tr>
<td>GBP</td>
<td>4.71% (0.462)</td>
<td>4.71% (0.462)</td>
<td>4.84% (0.691)</td>
</tr>
<tr>
<td>JPY</td>
<td>3.79% (0.001)</td>
<td>3.79% (0.001)</td>
<td>3.76% (0.001)</td>
</tr>
<tr>
<td>1/n No Short Min.</td>
<td></td>
<td>4.02% (0.01)</td>
<td>4.05% (0.013)</td>
</tr>
<tr>
<td>Variance Short Min.</td>
<td></td>
<td>4.15% (0.027)</td>
<td>4.25% (0.052)</td>
</tr>
<tr>
<td>Variance</td>
<td></td>
<td>4.58% (0.282)</td>
<td>4.55% (0.245)</td>
</tr>
<tr>
<td>Var 10% Failure Rates</td>
<td></td>
<td>8.93% (0.046)</td>
<td>8.86% (0.034)</td>
</tr>
<tr>
<td>EUR</td>
<td>8.60% (0.009)</td>
<td>8.43% (0.003)</td>
<td>8.50% (0.005)</td>
</tr>
<tr>
<td>GBP</td>
<td>8.93% (0.046)</td>
<td>9.00% (0.061)</td>
<td>9.52% (0.377)</td>
</tr>
<tr>
<td>JPY</td>
<td>7.97% (0.000)</td>
<td>7.97% (0.000)</td>
<td>8.01% (0.000)</td>
</tr>
<tr>
<td>1/n No Short Min.</td>
<td></td>
<td>8.60% (0.009)</td>
<td>8.60% (0.009)</td>
</tr>
<tr>
<td>Variance Short Min.</td>
<td></td>
<td>9.00% (0.061)</td>
<td>9.16% (0.011)</td>
</tr>
<tr>
<td>Variance</td>
<td></td>
<td>9.00% (0.061)</td>
<td>8.93% (0.046)</td>
</tr>
</tbody>
</table>

NOTE: The table reports the return failure rates, i.e., the percentage of time on which the realized loss exceed the predicted VaR, for each of the volatility models proposed. Failure rates for both individual currency returns and portfolios of currency returns are reported. VaR is predicted for the 1-day ahead and its level is set to \( \alpha = (0.01, 0.05, 0.1) \). The numbers in parenthesis are p-values of the Kupiec (1995) likelihood ratio test under the null hypothesis of correct coverage rate.
Figure 2.11: Forecast of $VaR_{0.01}$ for the Equally Weighted Portfolio

NOTE: The table reports the 1-day ahead forecast of the $VaR$ at $\alpha = 0.01$ for the equally weighted portfolio of currencies. The thin blue line denotes the mean of the predicted $VaR$ over the whole out-of-sample period.
Appendix to Chapter 2: Sample Data Excluded

The empirical analysis excludes data points corresponding to the following redefined days (from 21:05GMT of the previous calendar day to 21:00GMT):

- Saturdays and Sundays
- Christmas holidays: from December 24 to December 26
- New Year’s holidays: from December 31 to January 2
- US Moving holidays of Good Friday, Easter Monday, Memorial Day, Independence Day, Labor Day, Thanksgiving and the day after. Those are provided in the following table (format DD.MM.YYYY):

<table>
<thead>
<tr>
<th>Calendar Year</th>
<th>Good Friday</th>
<th>Easter +1</th>
<th>Memorial Day</th>
<th>Independence Day</th>
<th>Labor Day</th>
<th>Thanksgiving</th>
<th>Thanksgiving +1</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>06.04.2007</td>
<td>09.04.2007</td>
<td>28.05.2007</td>
<td>04.07.2007</td>
<td>03.09.2007</td>
<td>22.11.2007</td>
<td>23.11.2007</td>
</tr>
</tbody>
</table>
CHAPTER 3

The Volatility Asymmetry
Risk and Expected Returns

3.1 Introduction

A stylized fact in financial markets is the asymmetry in volatility. The volatility associated with bearish markets tends to be higher than the volatility associated with bullish markets. This results in a negative correlation between the dynamics of the stock price and volatility on average.

The volatility asymmetry has been thoroughly investigated by the extant literature and it has been traditionally explained with two compelling theories. According to the financial leverage hypothesis, a negative shock in the stock price leads the financial leverage to increase and, in turn, also the volatility, as it is an increasing function of the leverage. This view is supported, among others, by Christie (1982) and Schwert (1989). The other explanation is the feedback hypothesis. It asserts that an increase in volatility is associated with an expectation of higher future volatility and, therefore, market participants discount this information, resulting in an immediate drop in stock prices. The feedback theory has received extensive support in recent studies (see French et al., 1987; Campbell and Hentschel, 1992; Bekaert and Wu, 2000; Wu, 2001; Bae et al., 2007, among others).

I find that the sensitivity of stock returns to the market volatility asymmetry determines a statistically significant premium. This premium tends to be higher in magnitude for small, growth and high (beta) risk portfolios.
The implications of the volatility asymmetry to asset prices has been not
duly investigated by prior studies. This chapter fills this gap.

The volatility asymmetry is estimated using high frequency data. Specif-
ically, I estimate the market covariation between prices and volatilities on
each day. This high frequency estimator is the leverage effect of Mykland
and Zhang (2009) based on a continuous data generating process. There-
fore, it differs from the traditional interpretation of the leverage effect or the
volatility feedback effect in discrete time models, where prices and volatili-
ties have a lagged relation.

I assess separately the beta premium as due to the variation in the stock
market and to the variation in this new factor. In standard models for as-
set pricing, such as the CAPM, this asymmetry effect is not captured by
the market factor (excess return on the market portfolio) as it does not con-
tain information of its covariation with the volatility. The proposed pric-
ing model with asymmetric volatility factor performs well compared to the
CAPM with respect to different specification error tests. The new factor is
able to capture a small proportion of the cross sectional variation in stock
returns.

This chapter is related to the empirical asset pricing literature. The fail-
ure of the CAPM of Sharpe (1964), Lintner (1965), and Mossin (1966) to de-
scribe the average returns in the last four decades has led researchers to look
for new pricing models (for a survey, see Campbell, 2000) and for new fac-
tors that synthesize the systematic risks of an investment not captured by
the market portfolio. I propose the new factor within a linear pricing model.

The most closely related studies are Braun et al. (1995) and Cho and En-
gle (2000). They study beta asymmetry within a GARCH framework. Braun
et al. (1995) employ a multivariate EGARCH model for portfolio returns,
together with a stochastic process for the portfolio beta. The process for
the beta allows asymmetric responses to return shocks. They conclude that
there is no clear evidence for asymmetry in the beta premium as the coef-
cient for the asymmetry is statistically insignificant. In turn, most of the
volatility asymmetry observed both at portfolio level and market level is due to changes in the market volatility. Cho and Engle (2000) extend the work by using a double beta specification that captures both market and idiosyncratic risks. Moreover, they use a sample of daily return data. They find some evidence of asymmetric beta response to shocks. This work differs from previous papers in methodology as I first compute the asymmetric co-variation between prices and volatilities from the market and then estimate its effect on portfolio returns. Moreover, I use a wider data sample, and the analysis is conducted with different cross sections of portfolio returns formed on firm characteristics.

A related work by Ang and Chen (2002) investigates the asymmetric correlation between individual stocks and the aggregate market. They develop a statistics to measure the asymmetry based on exceedance correlation and they find a significant correlation asymmetry effect for small, growth and risky stocks. The present research differs from their work as I interpret the asymmetric beta effect not as asymmetric correlation between individual stock return and market return but as the sensitivity of individual stock return to the covariation between the market price and volatility. When the market itself displays asymmetric volatility this interpretation would be preferable as it allows the market volatility asymmetry to be priced.

This research is relevant for risk management and asset allocation. In a corporate perspective, the sensitivity to the market asymmetric volatility is a determinant for the cost of capital, and risk management practices should take into account this new risk factor. Portfolio managers could implement trading strategies, hedging for the asymmetric volatility, or mimic portfolios sorted by the level of volatility asymmetry.

The use of high frequency intra-day information in asset pricing tests is relatively new in the literature. Factor loadings constructed from high frequency data have been first considered by Bollerslev and Zhang (2003) for the Fama and French (1992) factors. Here, I focus on the daily volatility asymmetry factor by extrapolating information from high frequency intra-
day market prices.

The rest of this chapter is organized as follows. In the next section, I describe the estimator for the volatility asymmetry. Data and modeling strategies are presented in Section 3.3. The main empirical results, using daily data, are contained in Section 3.4. Finally, I conclude my findings in Section 3.5.

3.2 High Frequency Volatility Asymmetry

Recent developments in econometrics of high frequency data has permitted a further step in the analysis of security prices. The volatility asymmetry has been recently introduced by Mykland and Zhang (2009) and it was called as realized leverage effect. It is an estimator of the covariation between the volatility and the price of securities. Denoting with $X$ the log price, and $\sigma^2$ the volatility, the leverage effect for the period $T$ is

$$\text{ASY}_T = \langle \widehat{\sigma}^2, X \rangle_T = 2 \sum_{i=1}^{n} \left( \widehat{\sigma}^2_{\tau_{i+1}} - \widehat{\sigma}^2_{\tau_i} \right) (\Delta X_{\tau_i}), \quad (3.1)$$

with

$$\widehat{\sigma}^2_{\tau_i} = \frac{1}{\Delta t (M-1)} \sum_{t_j \in (\tau_i, \tau_{i+1}]} \left( \Delta X_{t_j} - \overline{\Delta X}_{\tau_i} \right)^2, \quad (3.2)$$

$$\overline{\Delta X}_{\tau_i} = \frac{1}{M} \sum_{t_j \in (\tau_i, \tau_{i+1}]} (\Delta X_{t_j}) = \frac{1}{M} (X_{\tau_{i+1}} - X_{\tau_i}). \quad (3.3)$$

The number of observations in each period $T$, representing here one day, is $n$ and the interval between two observations is $\Delta t = T/n$. The volatility is locally constant over the block size $M$, which represents the number of sub-intervals $(t_i, t_{i+1}]$ in each intra-day interval $(\tau_i, \tau_{i+1}]$. Equation (3.2) represents estimates of the local (intra-day) variance. The expression $\overline{\Delta X}_{\tau_i}$ is the average log return on the corresponding interval $(\tau_i, \tau_{i+1}]$ and it represents a small sample correction in estimating local variance.
3.2. High Frequency Volatility Asymmetry

The term leverage effect is attributed to the estimator for the covariation between prices and volatilities, \( \langle \sigma^2, X \rangle \), for practical reasons. In fact, this estimator does not imply any financial leverage on the underlying security. More properly, it measures the degree of asymmetry in the variation of the volatility associated with the variation of the log prices. When the log price process is a continuous semimartingale, this estimator is consistent. However, the presence of jumps in the price process is an obstacle as part of the estimated local variance is now the quadratic variation of jumps and therefore the estimates are asymptotically biased. In order to filter out the jump component in the return process I apply the test of Lee and Mykland (2008) to the raw series of returns. In the subsequent analysis the expression \( \Delta X \) represents the continuous return only, without the jump component, and the volatility asymmetry estimates the covariation between continuous variance and continuous returns. The appendix to this chapter describes the Lee and Mykland (2008) test for price jumps.

With respect to the literature on discrete time leverage and volatility feedback effects, in which the asymmetric effect between return and volatility is lagged, here, I document a lower level of negative asymmetry for high frequency contemporaneous covariation. In the sample period under consideration (from May 1982 to July 2010, 7061 daily observations), the daily volatility asymmetry is positive approximately 46.7% of the time and negative approximately 53.3% of the time. However, on average, the expression (3.1) tends to be negative, meaning that the cumulative variation in volatility tends to be higher for negative returns rather than positive returns. Consistently with prior research, the negative volatility asymmetry is more accentuated for high volatility values. In Campbell and Hentschel (1992), the volatility feedback effect is stronger for high levels of risk aversion. Bandi and Renò (2010) document a stronger leverage effect for high levels of volatility\(^1\). In addition, they also justify the time variation in the

\(^1\)Bandi and Renò (2010) propose an alternative non parametric estimator for the leverage effect. In their paper, the leverage effect is intended as the negative correlation between return
leverage effect, a characteristic of the data which is clear in this study. Figure 3.1 plots the time series of estimated daily volatility asymmetry.

Figure 3.1: Time Series of the Volatility Asymmetry

NOTE: The figure plots the time series of the estimated daily volatility asymmetry $ASY$, from April 1982 to August 2010.

The time series $ASY$ is very volatile, with most of days having an asymmetry value around zero and some days of very high positive or negative volatility asymmetry. Despite return jumps has been removed with the test of Lee and Mykland (2008), the volatility series appears to still include large jumps. Volatility asymmetry estimates are therefore not immune to outliers. A possible way to deal with the problem is to censor the estimated local volatility $\hat{\sigma}_T^2$, such that the resulting asymmetry series appears more smooth, or the $ASY$ series directly. However, neither volatility censoring nor asymmetry censoring is used in the analysis and those days with large and volatility differential.
volatility movements are not excluded from the sample data as I do not find a compelling economic rationale for exclusion of days with high volatility. Therefore, the regression results that follow are partly driven by those days with a large volatility asymmetry.

The time variation of the volatility asymmetry results from the stochastic price movements and a time-varying volatility. Under the assumption of constant volatility, the volatility asymmetry is zero (as the difference $\sigma_{i+1}^2 - \sigma_i^2$, in equation (3.1), equals zero, at any time $\tau$). Moreover, the volatility asymmetry, for the period $T$, is also zero if any positive comovement between price variation and volatility variation is offset by a negative covariation between prices and volatilities of the same magnitude.

### 3.3 Data, Sampling and Microstructure Issues

The data used in the analysis consists of tick-by-tick prices for the Standard & Poor (S&P500) Future, daily portfolio excess returns and Fama-French return factors. Intra-day prices for the S&P500 Future are used to compute the daily volatility asymmetry factor. The sample data I use covers almost the entire history of this security, from April 28, 1982 to August 6, 2010.

The S&P500 Future is traded “open outcry” on regular market opening hours of the Chicago Mercantile Exchange (from 8:30am to 15:15). The instrument is actively traded. I use the price of a traded security as proxy for the market price instead of a Cash Price Index, where the index is constructed by averaging the return of the 500 underlying securities. The price of an actively traded security is more volatile and it reflects better the investor reaction to common news. The S&P500 future dataset is also used in Bollerslev et al. (2009), among others, as an index for the composite market return.

Data on daily portfolio returns and control factors are obtained from

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2 As trades before 9:00AM for the first several year from the introduction of this security are often not very active, all the trades outside 9:00AM and 15:15 are discarded.
Prof. K. French’s website. I consider the 25 equally weighted portfolios formed on size and book-to-market value, the 30 and the 10 equally weighted portfolios formed on the industry category. All the portfolio returns are taken as excess returns with respect to 1-month T-Bill rate. The return data, together with the factors, are matched to estimated daily volatility asymmetry factor from the S&P500 Future (7061 daily observations).

I adopt the following sampling strategy for the estimation of the volatility asymmetry. I take market prices sampled every 5 minutes. As the theory on the inference for continuous processes is based on the notion of fixed time interval and decreasing sampling frequency, the time interval between two observations should be as small as possible. However, the empirical literature on realized variance commonly considers a fixed sampling scheme with price observations being measured every 5 minutes. By taking observation at lower frequency the impact of the microstructure noise is also lowered. The trade-off between having a sufficient number of observations to estimate the local variance and the daily asymmetric volatility effect depends also on the choice of the block size $M$. I fix the block size to 5, meaning that local volatility is assumed to be constant over an interval of time which is equal to $5 \cdot 5$ (minutes) = 25 (minutes). Therefore, only 5 return observations are used to estimate the local variance $\hat{\sigma}_i^2$, highlighting the importance of the small sample correction in equation (3.2).

While previous studies commonly use portfolio returns over arbitrary chosen quarterly, monthly, or weakly intervals, here, the choice to have daily return intervals in the analysis of asset prices is motivated by the following. It is argued by Levhari and Levy (1977) that the return interval in asset pricing models should correspond to the investment horizon and in the CAPM this investment horizon is assumed to be equal for all investors. In the management of risk, it is common practice for stock traders to rebalance their portfolios at the end of each trading day in order to have a risk exposure not exceeding a certain boundary. Also, with the development of new trading platforms and reduced trading fees, the investment horizon has certainly
been shortened with respect to the last three decades. This is even more stressed with strategies based on algorithmic trading, where portfolios are rebalanced in matter of seconds.

3.4 Empirical Evidence

3.4.1 Is the Volatility Asymmetry Priced?

To analyze the economical and statistical significance of the market leverage effect on expected returns, I run time series regression on portfolio returns. The following specification is employed:

\[ r_{i,t} = \alpha_i + \beta_i r_{M,t} + \gamma_i A\text{SYM}_{M,t} + \epsilon_{i,t}, \]  

(3.4)

where \( r_{i,t} \) is the excess return of portfolio \( i \) at time \( t \); \( r_{M,t} \) and \( A\text{SYM}_{M,t} \) are respectively the excess market return and the market volatility asymmetry. The error terms \( \epsilon_{i,t} \) are assumed to have zero expected value and constant conditional variance, and to be uncorrelated with the independent variables.

The regression is carried out on each individual return series of portfolios formed on predetermined characteristics. Grouping securities into portfolios is a common way of reducing the cross sectional dimension, with the benefit of smoothing the impact of the measurement error on individual securities. The selection of securities based on predetermined characteristics is also motivated empirically, in order to explain the cross sectional variation between portfolios with different characteristics. However, it is argued by Lo and MacKinlay (1990) that it induces a bias in asset pricing tests, generally known as data-snooping. Aware of this problem, the portfolio returns are not sorted by their asymmetric volatility ranking, in order to prevent any dependence of the market asymmetry factor on the preselected stock characteristic. Instead, I use the 25 Fama-French portfolios formed on size.
Chapter 3. The Volatility Asymmetry Risk and Expected Returns

and book-to-market value. I also conduct robustness tests with the 10 and 30 portfolio returns formed on the industry category (see Table 3.3)³.

Some linear dependencies exist between the different factors (see Table 3.1). More remarkably, the volatility asymmetry is negatively correlated with the continuous and jump volatilities. It is also negatively correlated with daily market return squared. Therefore, the sensitivity of average stock returns to the volatility asymmetry is partly related to the coskewness of Harvey and Siddique (2000). Due to this, pricing tests are performed by controlling for the different measures of the volatility level.

<table>
<thead>
<tr>
<th>Table 3.1: Factor Correlation Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ret_{Mkt}</td>
</tr>
<tr>
<td>Ret_{Mkt}</td>
</tr>
<tr>
<td>Ret^2_{Mkt}</td>
</tr>
<tr>
<td>CV_{Mkt}</td>
</tr>
<tr>
<td>JV_{Mkt}</td>
</tr>
<tr>
<td>ASY_{Mkt}</td>
</tr>
<tr>
<td>SMB</td>
</tr>
<tr>
<td>HML</td>
</tr>
</tbody>
</table>

NOTE: The table reports the pairwise correlation matrix between the different factors used in the analysis. "**" and "***" denote respectively significance at the 95% and 99% level.

Table 3.2 reports the results of the regressions (3.4). It also compares the results with the CAPM (with alpha) and it controls for the continuous volatility CV and jump volatility JV. The construction of those quantities are described in the appendix to this chapter. Inference based on the Newey-West heteroskedasticity and autocorrelation consistent standard errors yields statistically significant coefficient estimates for all the factors. The

³Time series regressions are reported for the 10 industry portfolios and not for the 30 industry portfolio in order to save space. Regression results for stocks sorted on 10 and 30 portfolios are similar and the conclusions are the same.
adjusted $R^2$ is reported as a measure of model performance. It gives indications that the specification with the volatility asymmetry factor is slightly better than the CAPM in explaining the time series variation of each expected portfolio return considered. The excess market return indeed captures most of the time series variation of average portfolio returns with very high fitting performance for large stock portfolios. The constant alpha is significantly different from zero for some of the portfolio returns, suggesting the presence of other pricing factors that are not accounted for.

Overall, the results confirm that the market volatility asymmetry is an additional source of risk. Small stocks are more exposed to this risk as the coefficients associated with it is higher in magnitude. This is consistent with the previous literature. Ang and Chen (2002) document stronger asymmetric beta effect for small and growth firms. When the covariation between market volatility and prices is negative, i.e., negative volatility asymmetry, the premium tends to be positive for small and growth portfolios and tends to be lower in magnitude and even negative for big and high value portfolios (see Table 3.2). This premium is also higher for high market beta portfolios with respect to low market beta (safer) investments (see Table 3.3). Therefore, stocks which are more exposed to the beta risk are also more exposed to the volatility asymmetry risk. By considering the control factors, the estimated coefficients associated to the volatility asymmetry do not change much, especially for portfolios in which they are statistically significant.

### 3.4.2 Hansen-Jagannathan Distance

I use the methodology of Hansen and Jagannathan (1997) to compare the specification errors of the different linear factor models. The $HJ$ distance is the least squares distance between the pricing kernel implied by the model specification and a set of correct pricing kernels. It also represents the maximum pricing error of a portfolio that has a unit second moment.
The stochastic discount factor (SDF) representation for asset prices in an economy without transaction costs follows the first order condition:

\[ E_{t-1} [R_t m_t] = 1, \]  

(3.5)

where \( R \) is the gross return of the asset and \( m_t \) is the SDF or pricing kernel. The linear factor pricing models are obtained with

\[ m_t = a + b' f_t, \]  

(3.6)

where \( f_t \) is a \((k \times 1)\) vector of factors at time \( t \), \( a \) is a constant and \( b \) is the vector of coefficients associated with the factors. I consider different specification of asymmetric volatility model by controlling for continuous and jump volatilities and other factors proposed in the literature. I compare the result with the traditional CAPM and Fama-French three factors model. All these models have unscaled factors and they include an intercept. They are specified as follows:

1. \( f_t = [R_{m,t}]'; \)
2. \( f_t = [R_{m,t} \ R^2_{m,t}]'; \)
3. \( f_t = [R_{m,t} \ ASY_{m,t}]'; \)
4. \( f_t = [R_{m,t} \ ASY_{m,t} \ CV_{m,t}]'; \)
5. \( f_t = [R_{m,t} \ ASY_{m,t} \ CV_{m,t} \ JV_{m,t}]'; \)
6. \( f_t = [R_{m,t} \ ASY_{m,t} \ R^2_{m,t}]'; \)
7. \( f_t = [R_{m,t} \ ASY_{m,t} \ R^2_{m,t} \ CV_{m,t}]'; \)
8. \( f_t = [R_{m,t} \ SMB_t \ HML_t]'; \)
9. \( f_t = [R_{m,t} \ ASY_{m,t} \ SMB_t \ HML_t]'; \)

The pricing errors that arise for each of the models are given by

\[ e_t = E [R_t m_t] - 1_N, \]  

(3.7)

and the squared HJ distance takes the form
\[ \delta^2 = \mathbb{E} \left[ e_t (\lambda) \right]' V^{-1} \mathbb{E} \left[ e_t (\lambda) \right], \]  
\[ (3.8) \]

with \( V = \frac{1}{T} \sum_{t=1}^{T} R_t R_t' \) being the sample covariance matrix of returns and \( \lambda = [a' b']' \) being the vector of parameters that minimizes \( \delta^2 \). \( T \) represents here the whole sample period and \( N \) represents the cross-sectional dimension. By defining \( x = [1_T f'] \) and \( D = \mathbb{E} [Rx] = \frac{1}{T} \sum_{t=1}^{T} R_t x_t' \), Kan and Robotti (2009) show that the least squares distance is estimated with

\[ \hat{\delta}^2 = \mathbb{I}_N' V^{-1} \mathbb{I}_N - \mathbb{I}_N' V^{-1} D \left( D' V^{-1} D \right)^{-1} D V^{-1}, \]  
\[ (3.9) \]

and the least squares estimate for the vector of parameters is

\[ \hat{\lambda} = \left( D' V^{-1} D \right)^{-1} \left( D' V^{-1} \mathbb{I}' \right). \]  
\[ (3.10) \]

Following Kan and Robotti (2009), I compute the standard error for \( \hat{\delta} \) under the assumption that the asset pricing models are misspecified. The asymptotic distribution of \( \delta \) is studied in Hansen and Jagannathan (1997) and Hansen et al. (1995). Specifically, when \( \delta^2 \neq 0 \),

\[ \sqrt{T} \left( \delta^2 - \delta^2 \right) \xrightarrow{d} \left( 0, \sigma^2 \right), \quad \text{as} \quad T \to \infty. \]  
\[ (3.11) \]

The asymptotic variance \( \sigma \) is estimated with the time series sequence of distances:

\[ d_t = m_t^2 - (m_t - \eta' R_t)^2 - 2\eta' \mathbb{I}_N - \delta^2, \]  
\[ (3.12) \]

with \( \eta = V^{-1} e \) and \( \eta' \mathbb{I}_N = e' V^{-1} (D \lambda - e) = -\delta^2 \). As the time sequence \( d_t \) displays some autocorrelation, I use the spectral density estimator of Newey and West (1987) to compute the standard errors of \( \hat{\delta} \) (using 15 lags as in Hansen and Jagannathan, 1997).

Under the assumption of a correctly specified model, \( \delta^2 = 0 \), Jagannathan and Wang (1996) show that \( T \delta^2 \) follow a weighted \( \chi^2 \) distribution:
\( T \hat{\delta}^2 \overset{d}{\longrightarrow} \sum_{i=1}^{N-k-1} \xi_i v_i \quad \text{as} \quad T \to \infty, \)  

(3.13)

where \( v_i \) are independent \( \chi^2_1 \) random variables and the weights \( \xi_i \) are non-zero eigenvalues of the matrix

\[
A = S^{1/2}V^{-1/2} \left[ I_N - (V^{-1/2})' D (D'V^{-1}D)^{-1} D'V^{-1/2} \right] (V^{-1/2})' (S^{-1/2})',
\]

(3.14)

where \( I_N \) is the identity matrix. \( S^{1/2} \) and \( V^{-1/2} \) are the upper-triangular matrices from the Cholesky decomposition of \( S = \frac{1}{T} \sum_{t=1}^{T} e_t e_t' \) and \( V^{-1} \), respectively. The random variable is simulated 50,000 times to determine the Monte Carlo p-values for the \( HJ \) distance of each model specification.

Table 3.4 provides results of the Hansen-Jagannathan test with respect to the different models considered and to cross sections of size/book-to-market and industry sorted portfolios. The estimated \( HJ \) square distance and the associated Newey-West standard errors are computed under the hypothesis that the models are misspecified. The \( p \)-value of the \( HJ \) test under the null \( H_0 : \delta^2 = 0 \) is denoted with \( P (\hat{\delta}^2 = 0) \).

Most of the models considered are misspecified. The \textit{CAPM}, the three factors Fama-French model and the Harvey and Siddique (2000) “3M” model are rejected at the 5% level. The previous results in Hansen and Jagannathan (1997) and Kan and Robotti (2009) report similar evidence, despite using monthly returns for portfolios. In those studies, the \( HJ \) distance was reported to be statistically and significantly different from zero for various linear factor models\(^4\). The results for model specification involving the volatility asymmetry factor differs between the cross sections of size/value sorted portfolios and the industry sorted portfolio differs. For the cross section of

\(^4\)Hansen and Jagannathan (1997) consider the scaled CAPM with equally weighted and value weighted market portfolio and the Breeden et al. (1989) consumption CAPM. Kan and Robotti (2009) consider scaled and unscaled versions of the CAPM, the consumption CAPM, the Jagannathan and Wang (1996) conditional CAPM, the Campbell (1996) intertemporal CAPM, the Fama-French three factors model and the Fama and French (1993) five factors model.
the 25 size/value sorted portfolios the test reject the null of zero pricing errors, while for the cross section of 30 industry sorted portfolio the test fails to reject the null of zero pricing errors. However combining the two cross sections of portfolios in one cross section of 55 portfolios the test appears to reject also all the model specifications with volatility asymmetry. The estimated sample $HJ$ distance, which is computed under the assumption of misspecified model, is a measure of model fit. It is substantially lower when the volatility asymmetry is included as a pricing factor. Therefore the volatility asymmetry factor has an incremental explanatory power also for a panel data of portfolio of returns.

3.4.3 Fama-MacBeth Regression

To examine the cross-sectional variation in expected returns I run the Fama and MacBeth (1973) regressions. I check whether the asymmetric volatility model specification helps to explain better the cross-section of average returns with respect to the $CAPM$, the “3M” co-skewness model and the Fama and French (1992) three factors model. Moreover, I test whether there is a positive price for the volatility asymmetry risk. Originally, the Fama-MacBeth test was proposed to test whether the market portfolio, in the $CAPM$, is efficient. Now, it is a standard methodology to assess the performance of an asset pricing model based on cross-sectional information.

The Fama-MacBeth procedure consists of the following two step regression with rolling betas. First, time series regressions are run to estimate the betas for each portfolio return. In the second step, the cross-section series of returns for each day $t$ is regressed on the estimated beta from the first step. In both steps, I allow for the presence of intercepts, the significance of which is an indication of whether the models are correctly specified.
The following two steps regression are employed:

\[ r_i = \alpha_i + X\beta_i + v_i, \quad (3.15) \]
\[ r_t = \delta_{0,t} + B_\|t\|\delta_t + \epsilon_t, \quad (3.16) \]

where \( r_i \) is the time series vector of return for portfolio \( i \), \( X \) is the matrix of pricing factors, \( \beta_i \) is the vector of betas (coefficients) for portfolio \( i \), \( B \) is the matrix of estimated betas for the different portfolio returns (on each row) and the different pricing factors (on each column), \( \delta_t \) is the vector of estimated coefficients for the second step cross-sectional regression at each day \( t \) and \( v_i \) and \( \epsilon_t \) are vectors of zero mean innovations. The subscript \( "|t" \) is added to the second step regression to denote that the betas in the first step regression are allowed to change over time, with daily rolling window regressions. The window is fixed to 1000 observations (corresponding approximately to four year), such that estimates for day \( t \) are obtained by using the sample data of the interval \((t - 1000, t]\). In the second step, \( r_t \), which is the vector of cross-sectional returns at day \( t \), is regressed on the corresponding estimated betas. Here, the estimation errors from the first step regression are ignored and the betas are taken as observed. As the second step regression requires betas estimated in the first step with a lagged sample of approximately four years, the cross-sectional regression starts from April 14, 1986. I consider the cross-section of the 25 portfolios sorted by size/book-to-market value and the 30 industry sorted portfolios. The Fama-MacBeth procedure results in 6062 rolling window time series regressions for each portfolio return, and also in 6062 cross sectional regressions. The least squares estimates of the coefficients \( \delta_j, j = 0, \ldots, k \), are averaged over the entire sample.

The following results can be pointed out from the Fama-MacBeth regressions. The cross-sectional intercept is significantly different from zero, indicating that all the models are misspecified. When the intercept is included in the cross-sectional regressions the price for the beta risk is estimated to
be negative\footnote{The paper of Petkova (2006, p. 598), among others, presents similar empirical evidence}. The \textit{CAPM} has a low performance in explaining the cross sectional variation of expected returns. The premium to the Fama-French size and value factors tends to be positive and contributes to a good proportion of the cross-sectional variation of average returns. The premium for the volatility asymmetry factor and the continuous and jump volatilities are positive and weakly statistically significant for the 25 size/value sorted portfolios but not for the 30 industry sorted portfolios. Overall, an increasing explanatory power, in terms of adjusted $R^2$ is attributed to the volatility asymmetry premium. Table 3.5 summarizes the result of the cross-sectional regressions for the six models and the two sets of portfolio cross-sections under consideration. It reports the estimated coefficients, the t-statistics computed with the method proposed by Fama and MacBeth (1973), and the average adjusted-$R^2$.

\section*{3.5 Conclusion}

The asymmetric volatility is an important characteristic of the stock market behavior. I show that a high frequency factor, which estimates this asymmetry, is able to explain a good proportion of the time variation and the cross-sectional variation of portfolio returns. The sensitivity of portfolio returns to the market volatility asymmetry yields a premium. Small, growth and high market beta stocks are particularly subject to this risk as investors require an additional premium for holding those stocks. Based on both Hansen-Jagannathan test and the Fama and MacBeth (1973) procedure, the volatility asymmetry contain a significant incremental power in explaining average portfolio returns, with a net improvement over the \textit{CAPM} in terms of model adjusted-$R^2$. The market price for the volatility asymmetry risk is positive and weakly statistically significant based on the Fama-MacBeth regressions.

The beta to the volatility asymmetry is another dimension to the concept of risk. This measure is highly volatile and a possible interpretation for it is
that of short term sentiment fluctuations of market participants. Also, this factor may be related to the battery of other factors proposed in the literature. It is partially related to the coskewness risk of Harvey and Siddique (2000). It may also be related to the macroeconomic risk factors of Chen et al. (1986), the “cay” of Lettau and Ludvigson (2001), the liquidity risk of Pástor and Stambaugh (2003), the volatility cross-section of Ang et al. (2006), and the financial distress risk of Garlappi and Yan (2011). In this study, I do not control for all those factors as, for most of those factors, data at daily frequency is not available.
NOTE: The table reports time series regression results for the 25 portfolio returns, sorted by size and book-to-market value. The pricing models are the CAPM with intercept, the CAPM plus the volatility asymmetry factor, denoted as \( ASY - CAPM \), before and after controlling for the continuous volatility and jump volatility, capturing market volatility levels, and daily market excess return squared, capturing co-skewness. Newey-West heteroskedasticity and autocorrelation consistent standard errors with 10 lags are reported in parentheses. The sample of daily return data ranges from April 28, 1982 to August 6, 2010.
Table 3.3: Regressions for Industry Sorted Portfolios

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>CAPM with alpha</th>
<th>Asymmetric CAPM (ASY-CAPM)</th>
<th>Controlling for CV and IV</th>
<th>Controlling for ( \beta_\text{Mkt}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Const.</td>
<td>Ret_Mkt</td>
<td>Adj-R²</td>
<td>Const.</td>
</tr>
<tr>
<td>Utilities</td>
<td>0.03</td>
<td>0.54</td>
<td>0.55</td>
<td>0.03</td>
</tr>
<tr>
<td>(0.007)</td>
<td>(0.028)</td>
<td></td>
<td></td>
<td>(0.007)</td>
</tr>
<tr>
<td>Consumer NonDurables</td>
<td>0.05</td>
<td>0.63</td>
<td>0.67</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.008)</td>
<td>(0.017)</td>
<td></td>
<td></td>
<td>(0.008)</td>
</tr>
<tr>
<td>Other</td>
<td>0.06</td>
<td>0.64</td>
<td>0.70</td>
<td>0.06</td>
</tr>
<tr>
<td>(0.007)</td>
<td>(0.02)</td>
<td></td>
<td></td>
<td>(0.007)</td>
</tr>
<tr>
<td>Shops</td>
<td>0.05</td>
<td>0.74</td>
<td>0.69</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.017)</td>
<td></td>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td>Healthcare</td>
<td>0.07</td>
<td>0.76</td>
<td>0.59</td>
<td>0.07</td>
</tr>
<tr>
<td>(0.013)</td>
<td>(0.014)</td>
<td></td>
<td></td>
<td>(0.013)</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.05</td>
<td>0.79</td>
<td>0.70</td>
<td>0.05</td>
</tr>
<tr>
<td>(0.009)</td>
<td>(0.026)</td>
<td></td>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td>Consumer Durables</td>
<td>0.03</td>
<td>0.80</td>
<td>0.62</td>
<td>0.03</td>
</tr>
<tr>
<td>(0.011)</td>
<td>(0.026)</td>
<td></td>
<td></td>
<td>(0.011)</td>
</tr>
<tr>
<td>Energy</td>
<td>0.06</td>
<td>0.82</td>
<td>0.39</td>
<td>0.06</td>
</tr>
<tr>
<td>(0.016)</td>
<td>(0.062)</td>
<td></td>
<td></td>
<td>(0.016)</td>
</tr>
<tr>
<td>Hi-Tech</td>
<td>0.06</td>
<td>0.92</td>
<td>0.66</td>
<td>0.06</td>
</tr>
<tr>
<td>(0.014)</td>
<td>(0.022)</td>
<td></td>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>Telecomunications</td>
<td>0.04</td>
<td>0.93</td>
<td>0.61</td>
<td>0.03</td>
</tr>
<tr>
<td>(0.013)</td>
<td>(0.022)</td>
<td></td>
<td></td>
<td>(0.013)</td>
</tr>
</tbody>
</table>

NOTE: The table reports time series regression results for the 10 industry category sorted portfolio returns. The pricing models are the CAPM with intercept, the CAPM plus the volatility asymmetry factor, denoted as \( ASY - CAPM \), before and after controlling for the continuous volatility and jump volatility, capturing market volatility levels, and daily market excess return squared, capturing co-skewness. Newey-West heteroskedasticity and autocorrelation consistent standard errors with 10 lags are reported in parentheses. The sample of daily return data ranges from April 28, 1982 to August 6, 2010.
### Table 3.4: Hansen-Jagannathan Distance

<table>
<thead>
<tr>
<th>Portfolio Cross-section</th>
<th>Model</th>
<th>HJ $\delta^2$</th>
<th>St.Err.($\delta^2$)</th>
<th>P($\delta^2 = 0$)</th>
<th>n. factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>25 Size - Value</td>
<td>CAPM (alpha)</td>
<td>0.407</td>
<td>0.072</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3M</td>
<td>0.324</td>
<td>0.069</td>
<td>0.002</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>ASY-CAPM</td>
<td>0.392</td>
<td>0.069</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>ASY-CV</td>
<td>0.334</td>
<td>0.057</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-CJV</td>
<td>0.323</td>
<td>0.054</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>ASY-3M</td>
<td>0.287</td>
<td>0.064</td>
<td>0.009</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-3M-CV</td>
<td>0.278</td>
<td>0.055</td>
<td>0.003</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>FF</td>
<td>0.381</td>
<td>0.07</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-FF</td>
<td>0.376</td>
<td>0.069</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>30 Industry</td>
<td>CAPM (alpha)</td>
<td>0.658</td>
<td>0.182</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3M</td>
<td>0.623</td>
<td>0.192</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>ASY-CAPM</td>
<td>0.462</td>
<td>0.18</td>
<td>0.253</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>ASY-CV</td>
<td>0.453</td>
<td>0.184</td>
<td>0.196</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-CJV</td>
<td>0.414</td>
<td>0.194</td>
<td>0.288</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>ASY-3M</td>
<td>0.397</td>
<td>0.2</td>
<td>0.366</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-3M-CV</td>
<td>0.384</td>
<td>0.196</td>
<td>0.424</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>FF</td>
<td>0.355</td>
<td>0.131</td>
<td>0.045</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-FF</td>
<td>0.321</td>
<td>0.134</td>
<td>0.33</td>
<td>4</td>
</tr>
<tr>
<td>10 Industry</td>
<td>CAPM (alpha)</td>
<td>0.52</td>
<td>0.149</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>3M</td>
<td>0.518</td>
<td>0.147</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>ASY-CAPM</td>
<td>0.331</td>
<td>0.16</td>
<td>0.004</td>
<td>2</td>
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<tr>
<td></td>
<td>ASY-CV</td>
<td>0.323</td>
<td>0.153</td>
<td>0.001</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-CJV</td>
<td>0.158</td>
<td>0.185</td>
<td>0.409</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>ASY-3M</td>
<td>0.294</td>
<td>0.156</td>
<td>0.012</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-3M-CV</td>
<td>0.203</td>
<td>0.117</td>
<td>0.06</td>
<td>4</td>
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<tr>
<td></td>
<td>FF</td>
<td>0.115</td>
<td>0.075</td>
<td>0.02</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>ASY-FF</td>
<td>0.092</td>
<td>0.065</td>
<td>0.022</td>
<td>4</td>
</tr>
</tbody>
</table>

**NOTE:** The table reports the estimated Hansen-Jagannathan squared distance, $\delta^2$, for different model specifications and for portfolios sorted by size/value and industry categories. The standard errors are computed under the assumption that the models are misspecified ($\delta^2 \neq 0$) with the method described in Newey and West (1987) with 15 lags. $P(\delta^2 = 0)$ represents the Monte Carlo p-value for the misspecification test under the null $\delta^2 = 0$. Return data used to compute the $HJ$ distance are in percentage.
Table 3.5: Fama-MacBeth Regressions

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Model</th>
<th>$\delta^{\times 10}$</th>
<th>$\delta_0$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\delta_3$</th>
<th>$\delta_4$</th>
<th>$\delta_5$</th>
<th>$\delta_6$</th>
<th>$\delta_8$</th>
<th>Adj-$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAPM (alpha)</td>
<td></td>
<td>1.13</td>
<td>-0.87</td>
<td>[11.48]</td>
<td>[-5.06]</td>
<td></td>
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NOTE: The table reports results of the Fama-MacBeth cross-sectional regressions for portfolios sorted by size/value and industry categories. The cross sectional regression is conducted for each day of the sample period. The adjusted-$R^2$ and the coefficients reported represent the average value for, respectively, the adjusted-$R^2$ and the coefficients on each day over the entire sample (from April 14, 1986 to August 6, 2010). All the coefficients are multiplied by 10. In brackets, t-statistics, based on the methodology proposed by Fama and MacBeth (1973), are reported.
Appendix to Chapter 3: Test for Jumps (Lee and Mykland, 2008) and Identification of Continuous and Jump Volatility

To identify return jumps the Lee and Mykland (2008) test is employed. The test statistic is based on the ratio between intraday returns and estimates of the spot volatility:

$$
L_{t,i} = \frac{r_{t,i}}{\left(\frac{1}{K-1} \sum_{k=1}^{K-1} \left| r_{t,i-k} \right| \left| r_{t,i-(k-1)} \right| \right)^{1/2}}.
$$

(3.17)

The expression contains in the denominator the estimates of the spot volatility as the average bipower volatility over a period with $K$ observations. The analysis uses $K = 270$ as suggested by Lee and Mykland (2008) with return sampled at a frequencies of 5 minutes. Under the null of no intraday jump, the test statistics $L_{t,i}$ has asymptotically a normal distribution, with variance $\mu^2 = \frac{2}{\pi}$, where $\mu$ is the scaling constant used to estimate the bipower variation of Barndorff-Nielsen and Shephard (2004b):

$$
BV_t = \mu^{-2} \sum_{i=2}^{n} \left| r_{t,i} \right| \left| r_{t,i-1} \right|.
$$

(3.18)

As noted by Lee and Mykland (2008), a t-test would detect too many “spurious” jumps every day. In order to reduce the detection of false jumps they propose to look at the asymptotic distribution of maxima of the test statistics. As the sampling frequency tends to zero, under the null of no jumps between time $(t, i-1)$ and $(t, i)$, the absolute value of $L_{t,i}$ converges to a Gumbel distribution:

$$
\max_{(t,j)} \left| L_{t,i} \right| - C_n \xrightarrow{d} \zeta,
$$

(3.19)

where $\zeta$ has a standard Gumbel distribution, $C_n = \frac{(2 \log n)^{1/2}}{\mu} - \frac{\log \pi + \log(\log n)}{2\mu(2 \log n)^{1/2}}$. 
\[ S_n = \frac{1}{\mu (2 \log n)^{1/2}}, \text{ and } n \text{ is the number of observations for each period } t. \]

The test reject the null of no jump at time \((t, i)\) if

\[ \frac{|\mathcal{L}_{t,i}| - C_n}{S_n} > \beta^*, \tag{3.20} \]

such that \(\exp \left( -e^{-\beta^*} \right) = 1 - \alpha\), i.e. \(\beta^* = -\log (\log (1 - \alpha))\), with \(\alpha\) being the significance level.

The test is able to detect the jump arrival time \(i_j\) for each day \(t\), where \(j\) denotes the presence of a jump. Moreover the jump size is computed as

\[ \kappa_{t,i} = (r_{t,i}) \mathbb{1}\left\{ \frac{|\mathcal{L}_{t,i}| - C_n}{S_n} \leq \beta^* \right\}. \tag{3.21} \]

Daily jump volatility is computed as the square root of the sum of squared intraday jumps:

\[ JV_t = \left( \sum_{i_j = i_1, \ldots, i_T} \kappa_{t,i_j}^2 \right)^{1/2}, \quad t = 1, \ldots, T. \tag{3.22} \]

Cosequently, continuous intraday returns are only the remaining intraday returns which are not jumps,

\[ cRet_{t,i} = (r_{t,i}) \mathbb{1}\left\{ \frac{|\mathcal{L}_{t,i}| - C_n}{S_n} > \beta^* \right\}, \quad t = 1, \ldots, T, \tag{3.23} \]

and continuous volatility is computed as

\[ CV_t = \left( \sum_{i=1}^{n} cRet_{t,i}^2 \right)^{1/2}, \quad t = 1, \ldots, T. \tag{3.24} \]

The realized variation is equal to the sum of continuous and jump variations and therefore the realized volatility is equal to

\[ RV = \left( CV_t^2 + JV_t^2 \right)^{1/2}. \tag{3.25} \]


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